System Design and Methodology / Embedded Systems Design

IV. Petri Nets

TDTS07/TDDI08
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(Based on material by Petru Eles and Soheil Samii)

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PETRI NETS

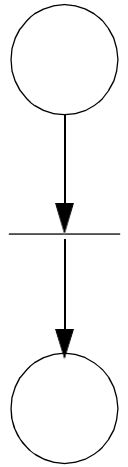
1. Basic Petri Net Model

2. Properties and Analysis of Petri Nets

3. Extended Petri Net Models

Petri Nets

- Systems are specified as a directed bipartite graph.
  The two kinds of nodes in the graph:



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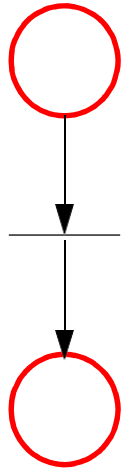

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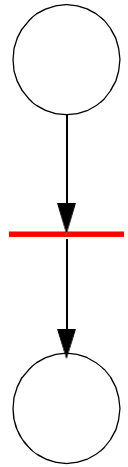

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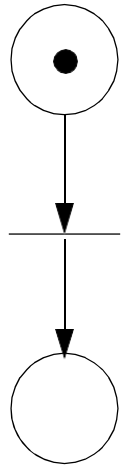
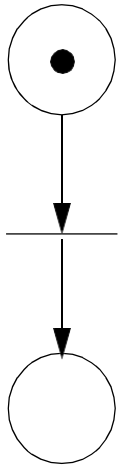


- *The state of the system*: captured by the marking of the places (number of tokens in each place)

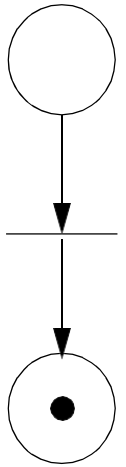
Petri Nets

- The dynamic evolution of the system: determined by the firing process of transitions.
  - A transition is enabled and may fire whenever all its predecessor places are marked.



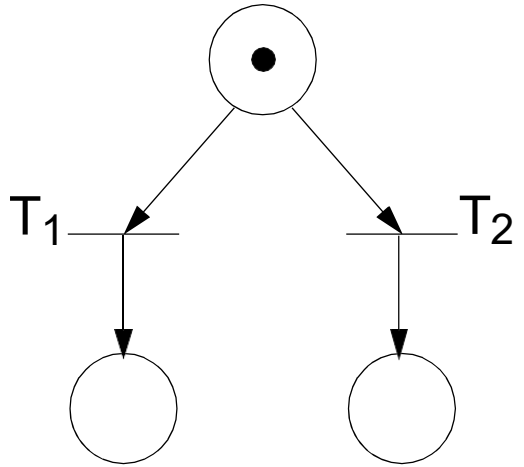
Petri Nets

- The dynamic evolution of the system: determined by the firing process of transitions.
  - A transition is enabled and may fire whenever all its predecessor places are marked.
  - If a transition fires it removes a token from each predecessor place and adds a token to each successor place.



Nondeterminism

- Both T1 and T2 are enabled and any of the two may fire.



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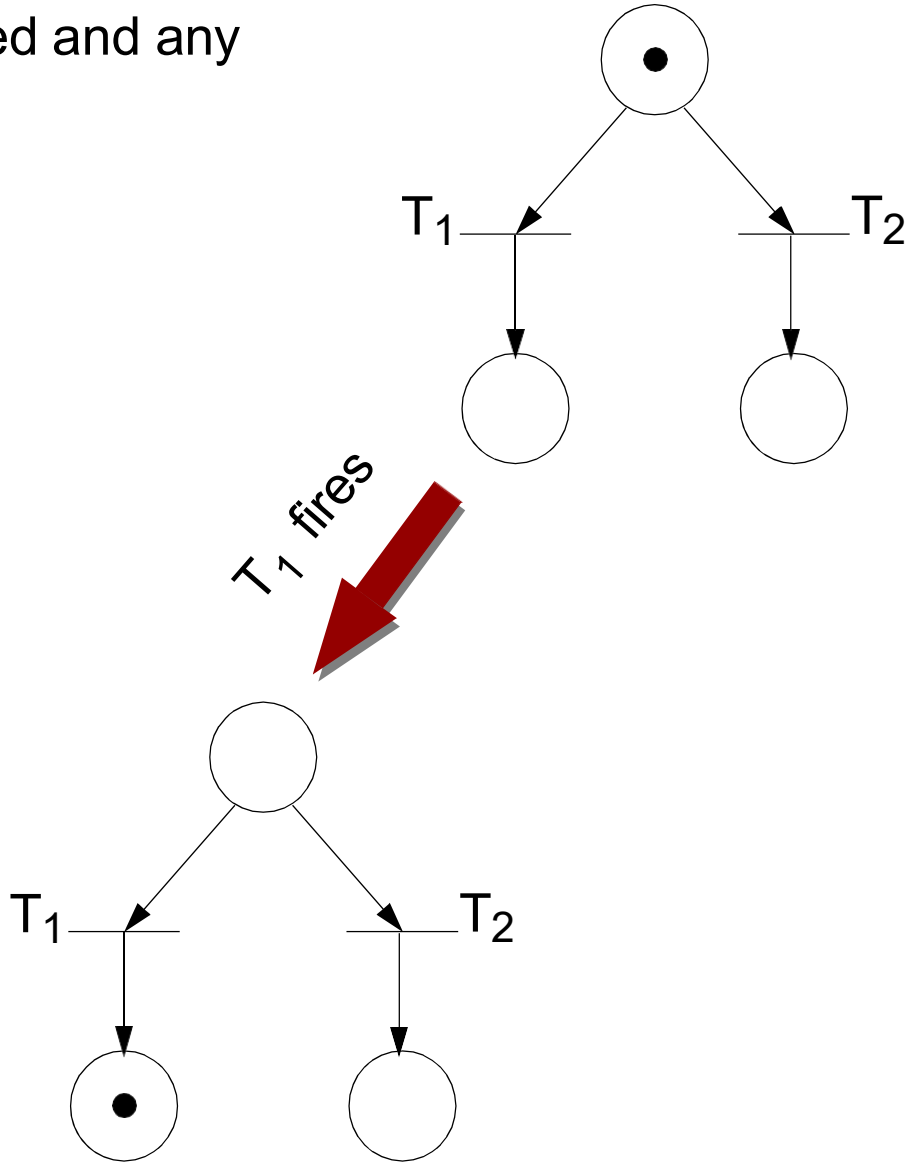

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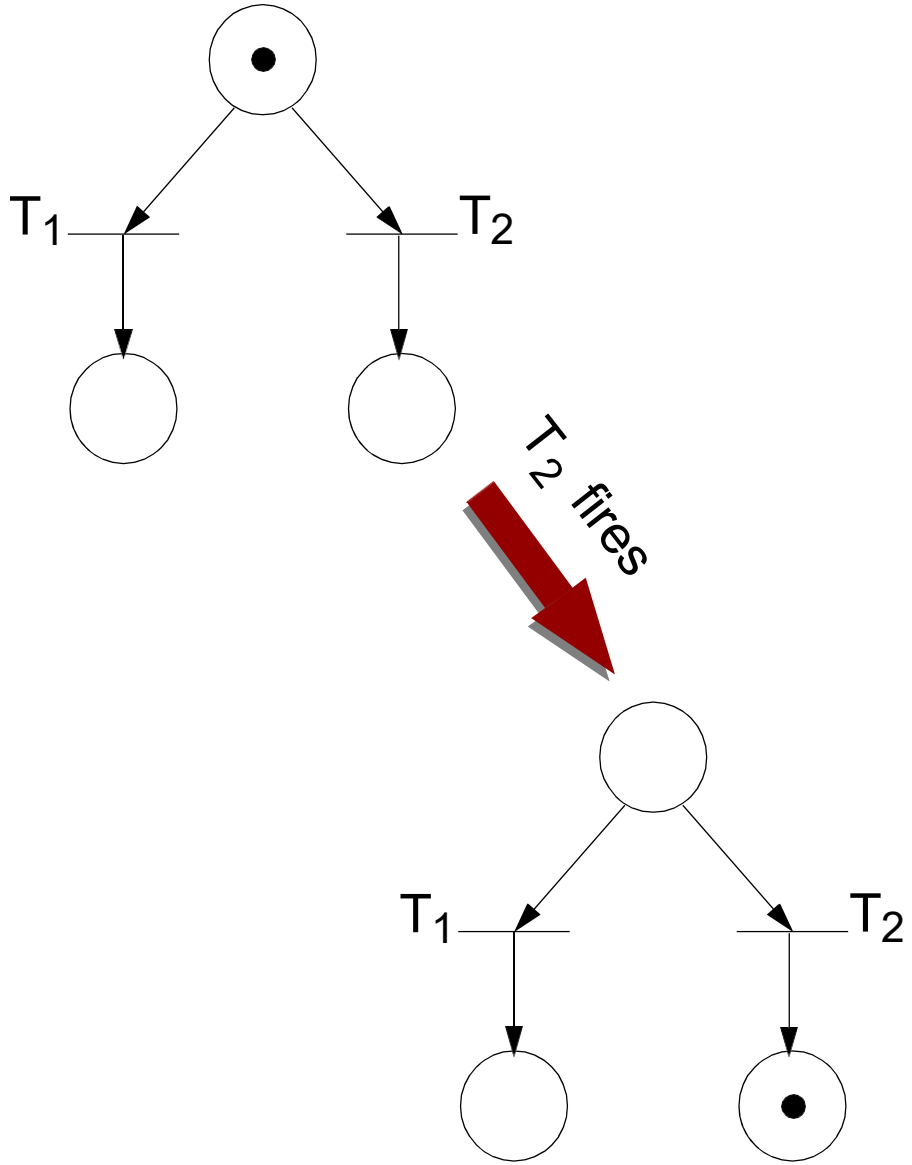

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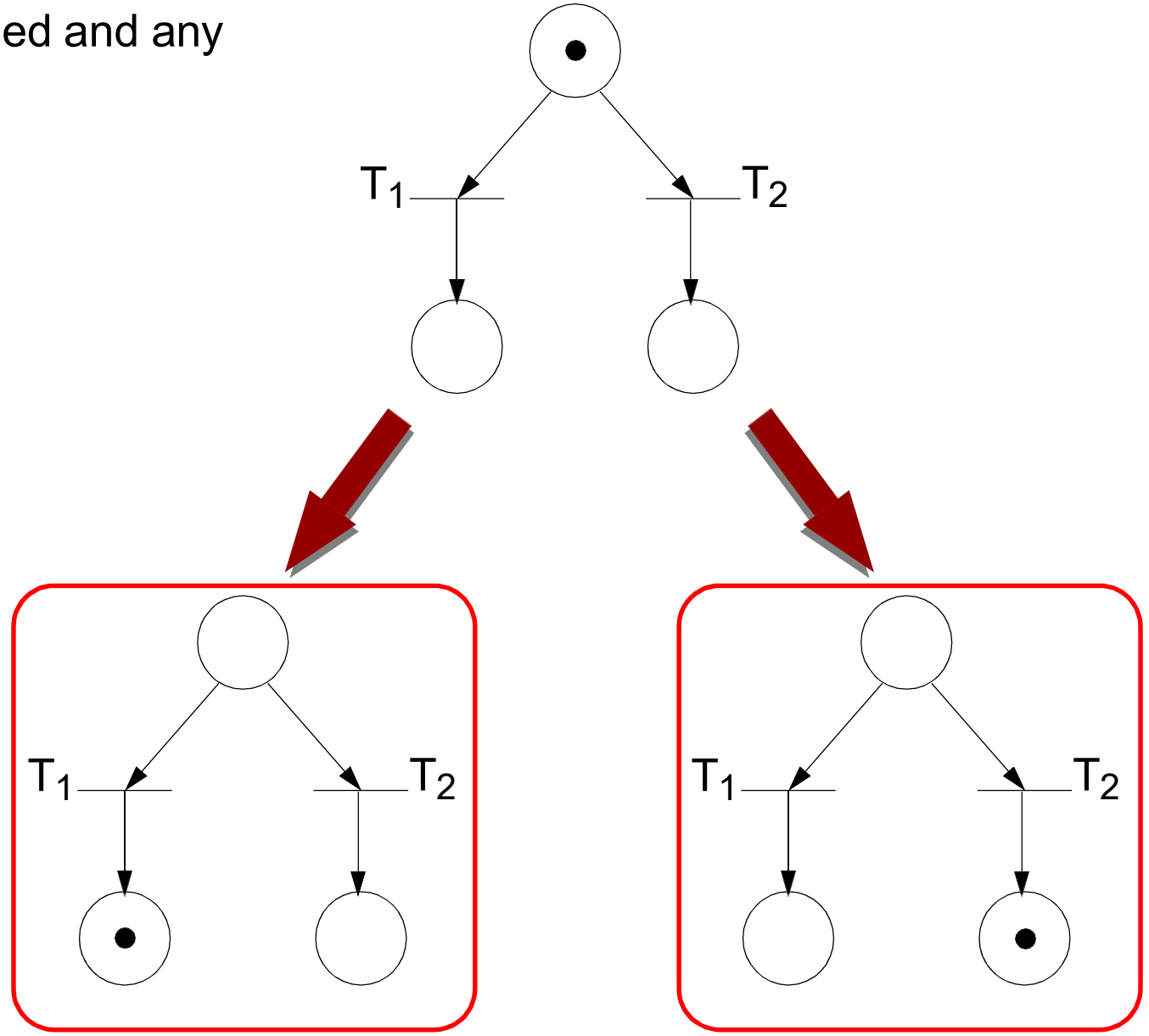

- Any of these two states might be the next state.

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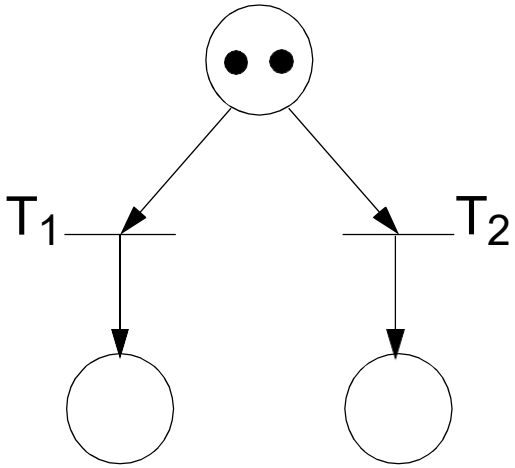

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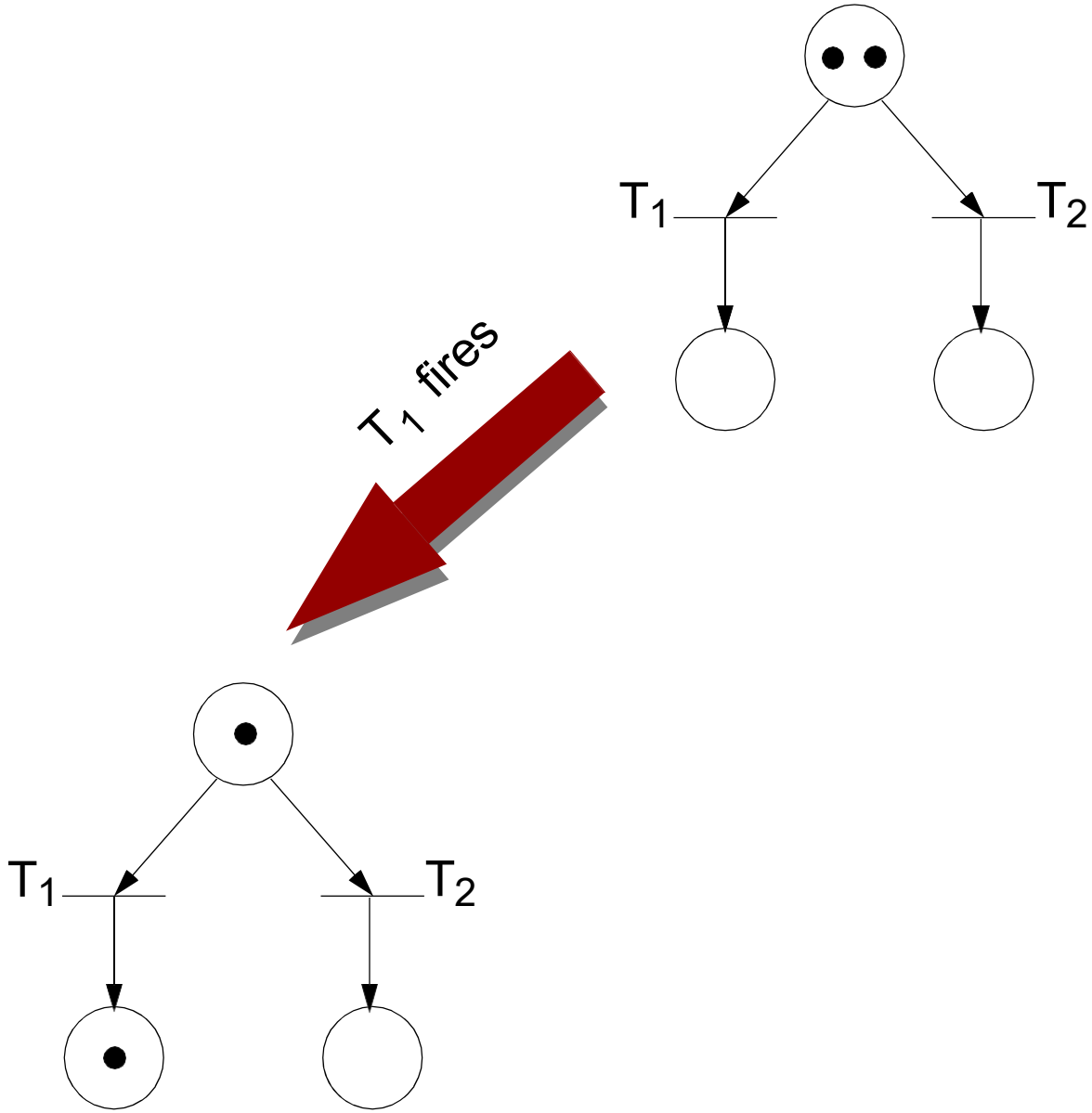

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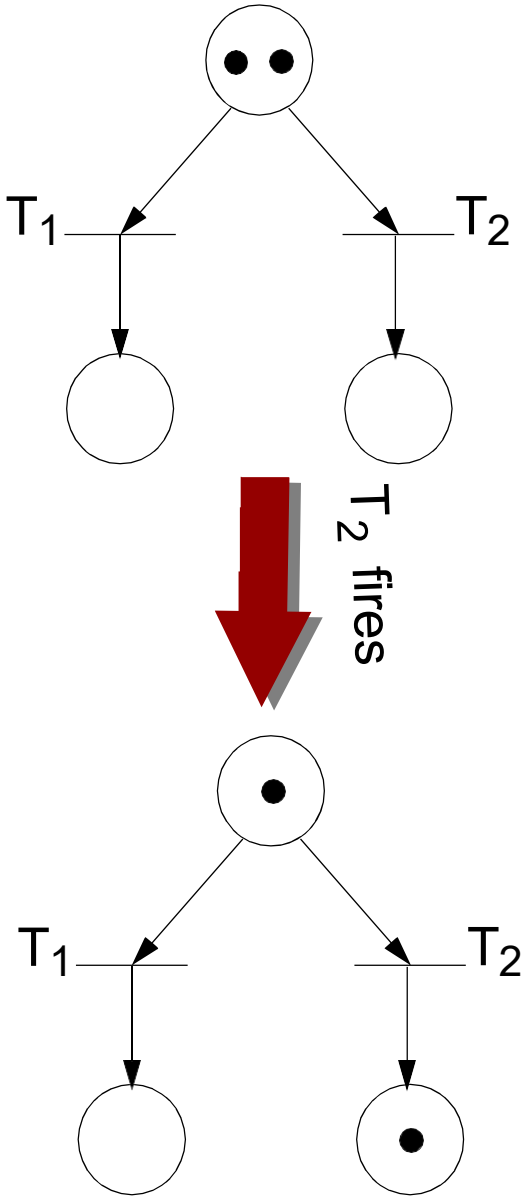

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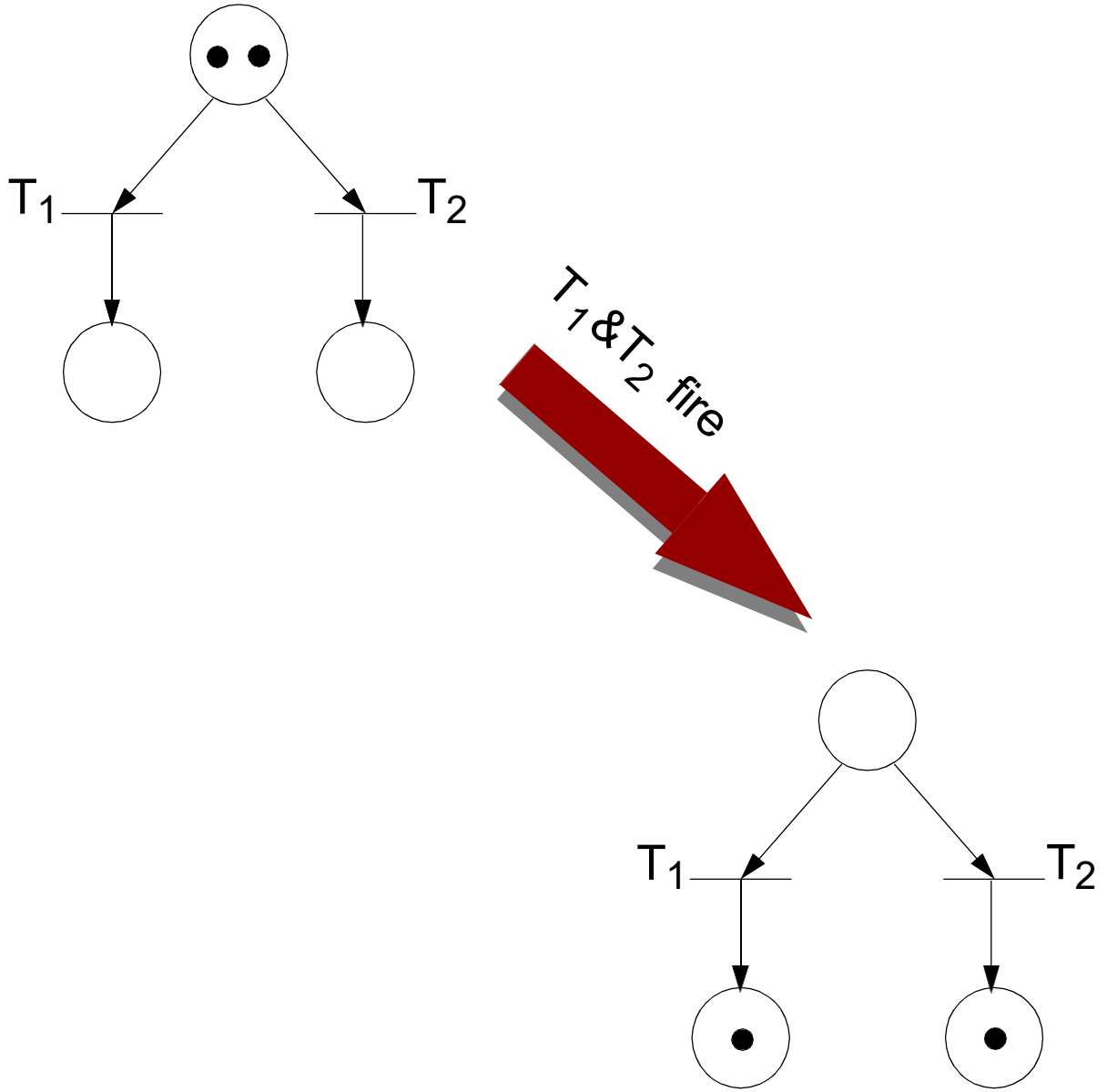

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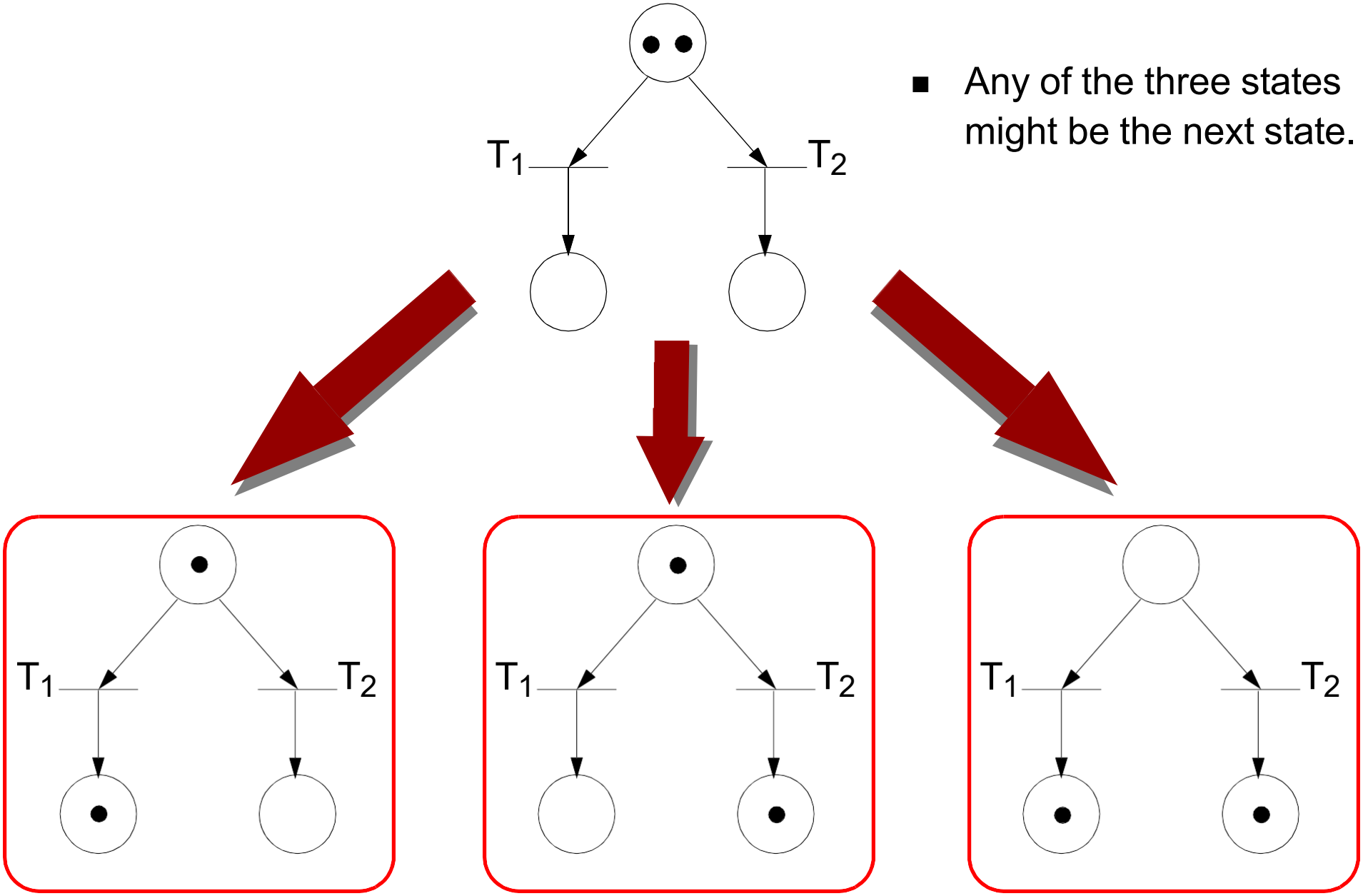

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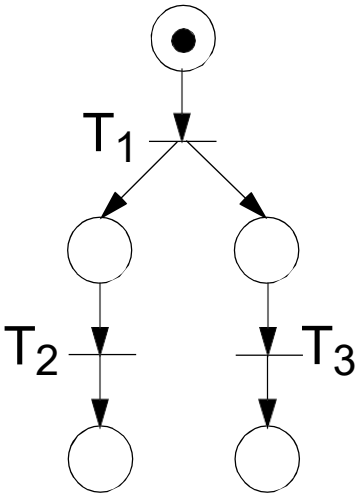
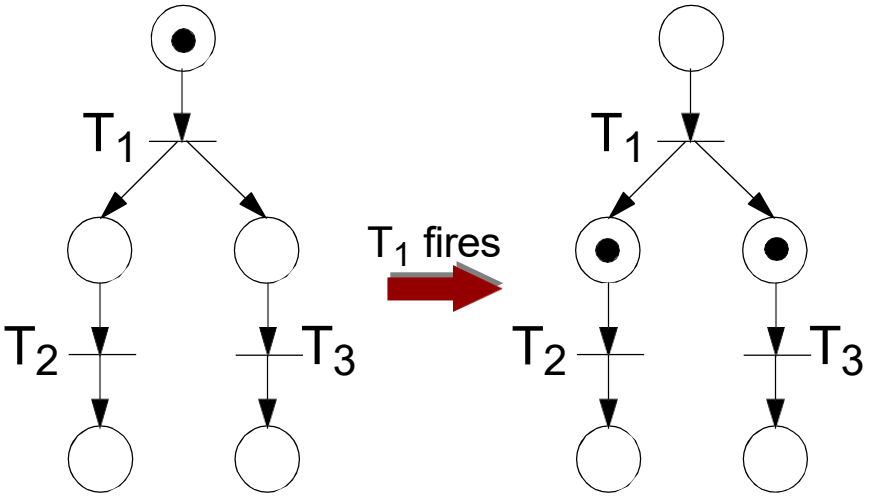


Nondeterminism

- No nondeterminism here:
  T_1 is the only enabled transition!



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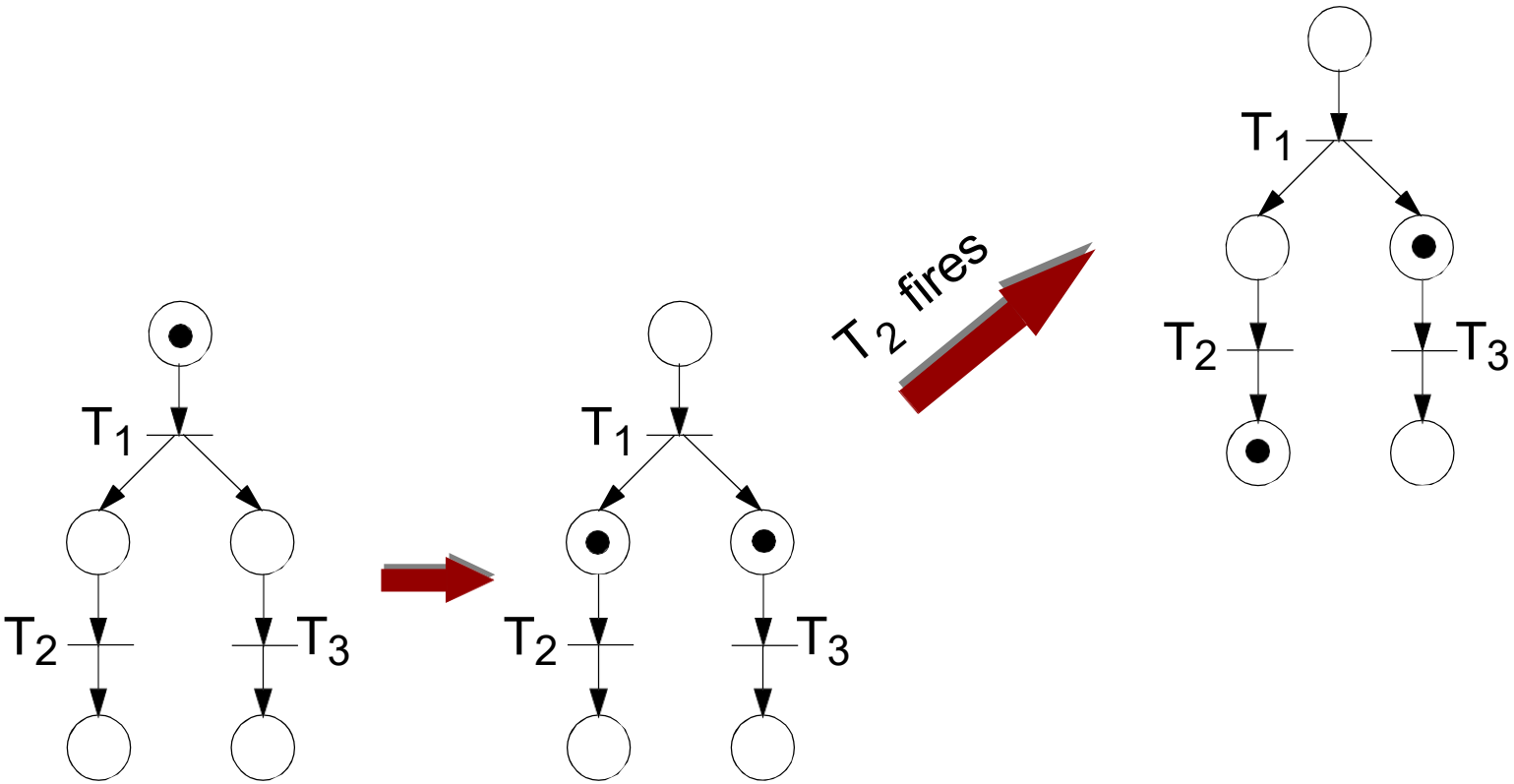

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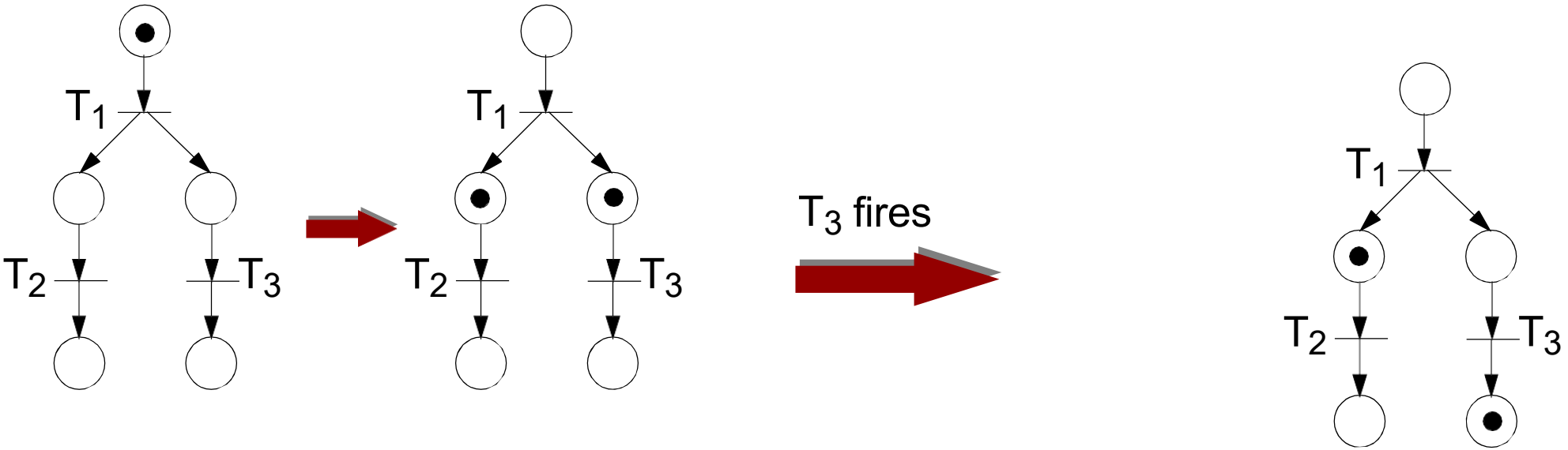

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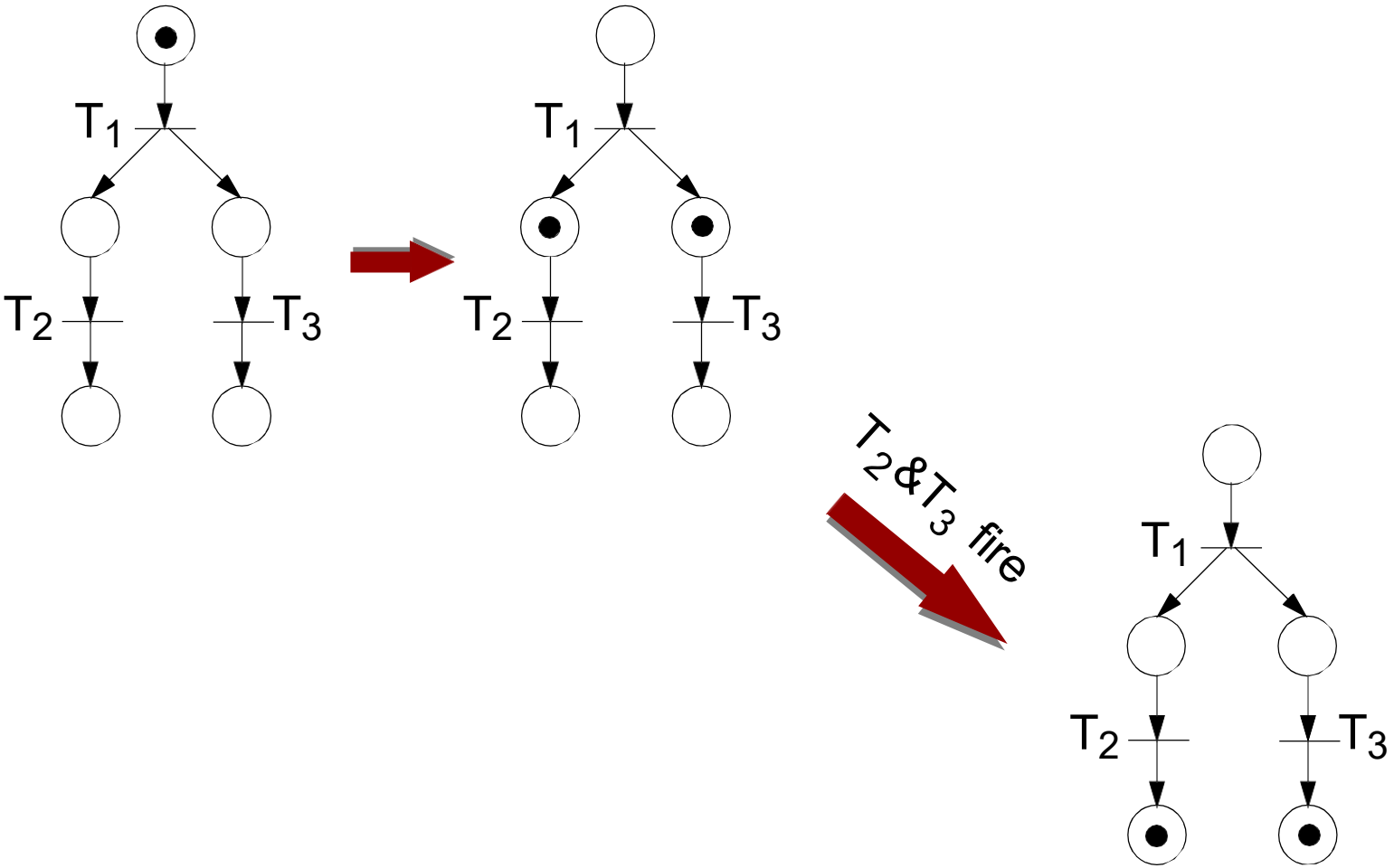

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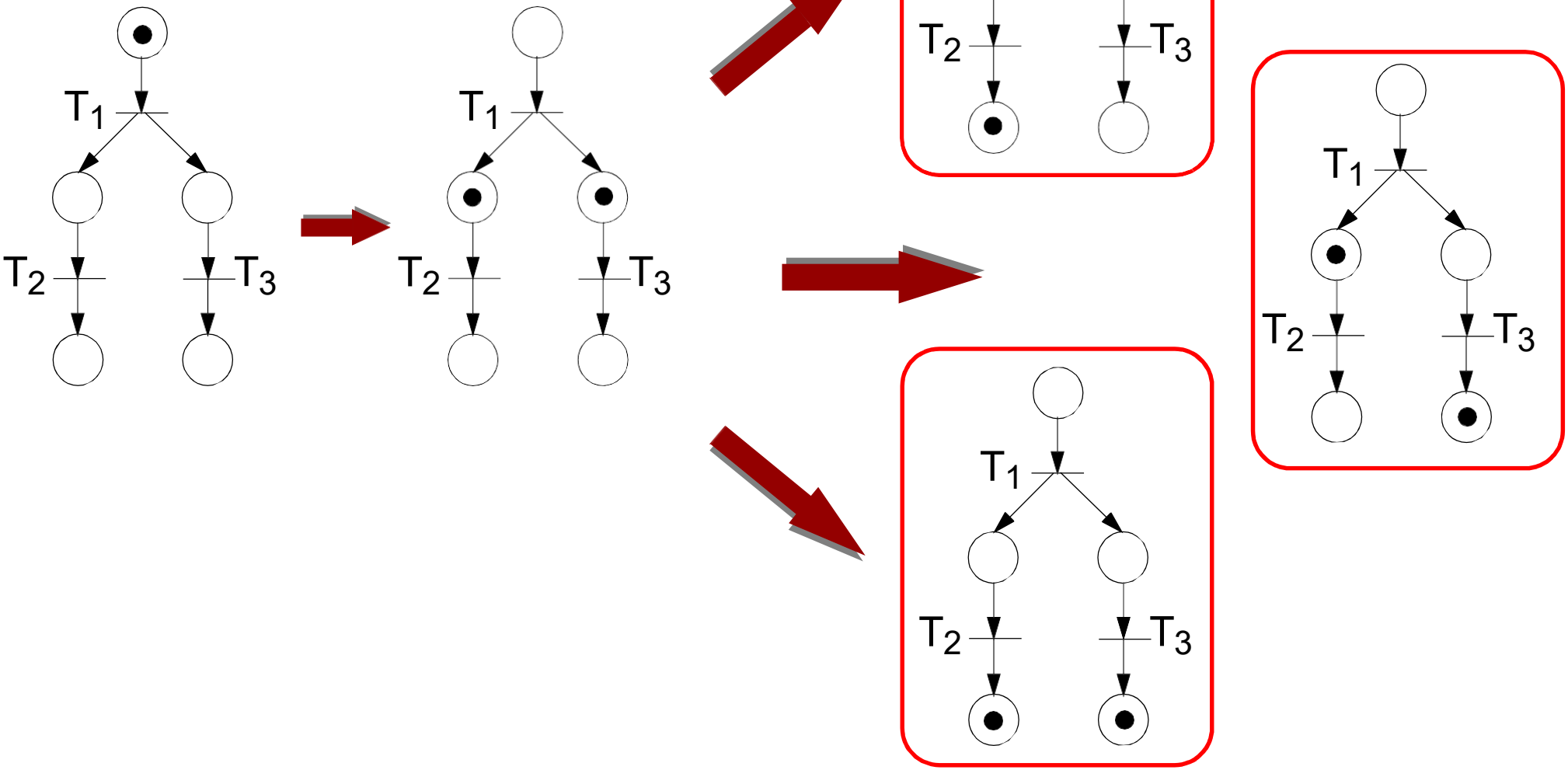

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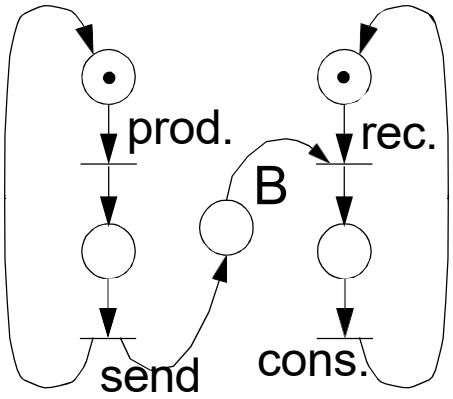
Petri Net Example

A producer and a consumer process communicating through a buffer:



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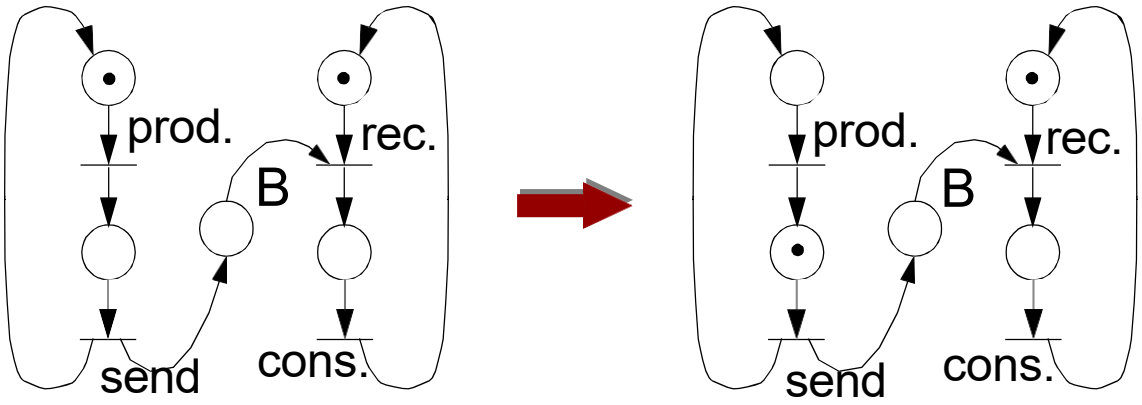

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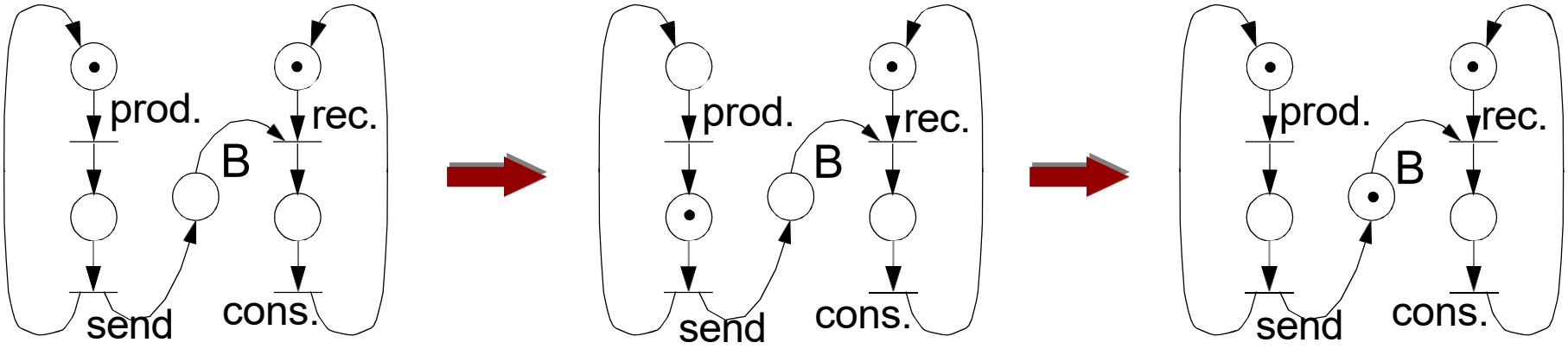

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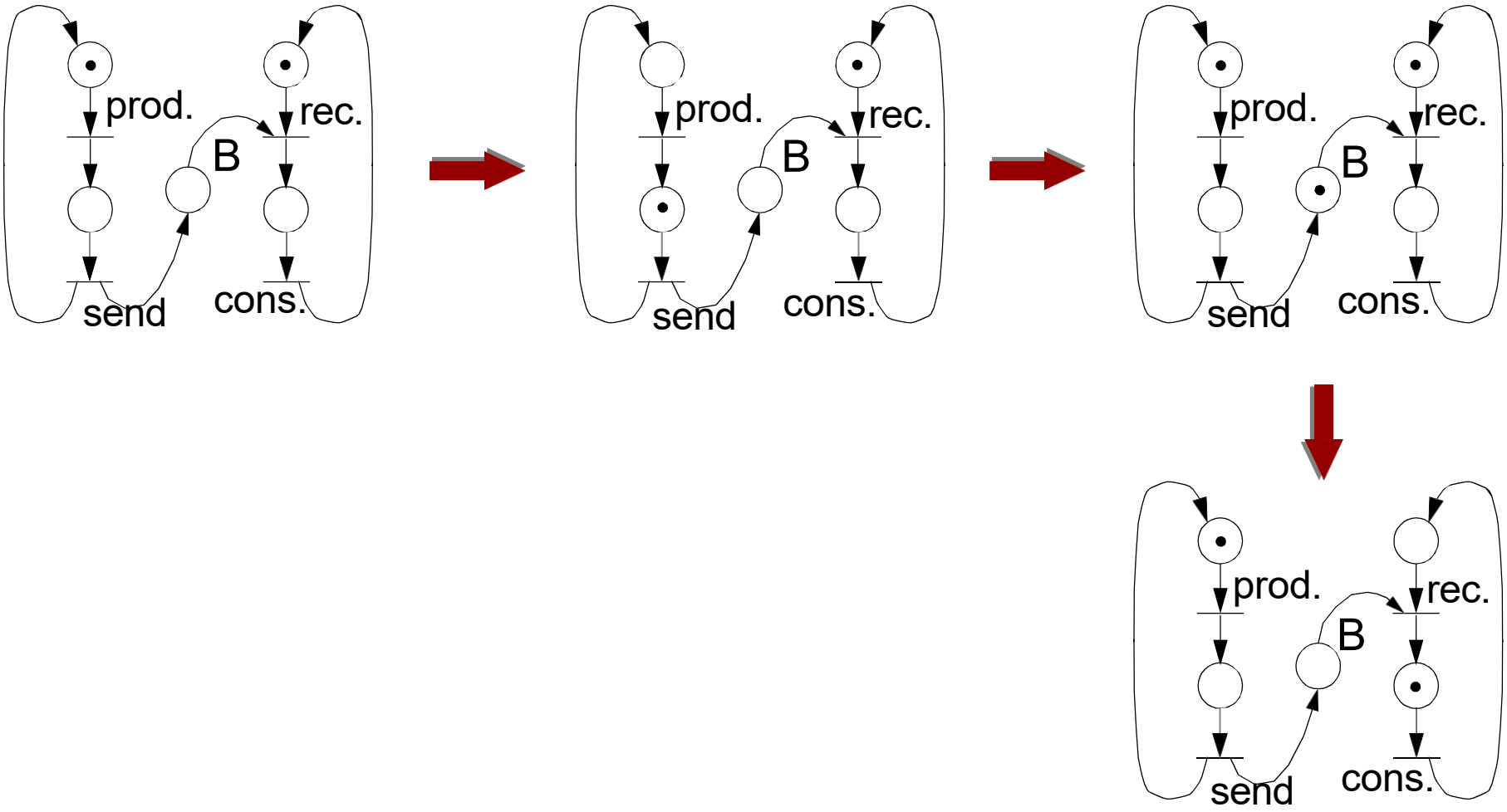

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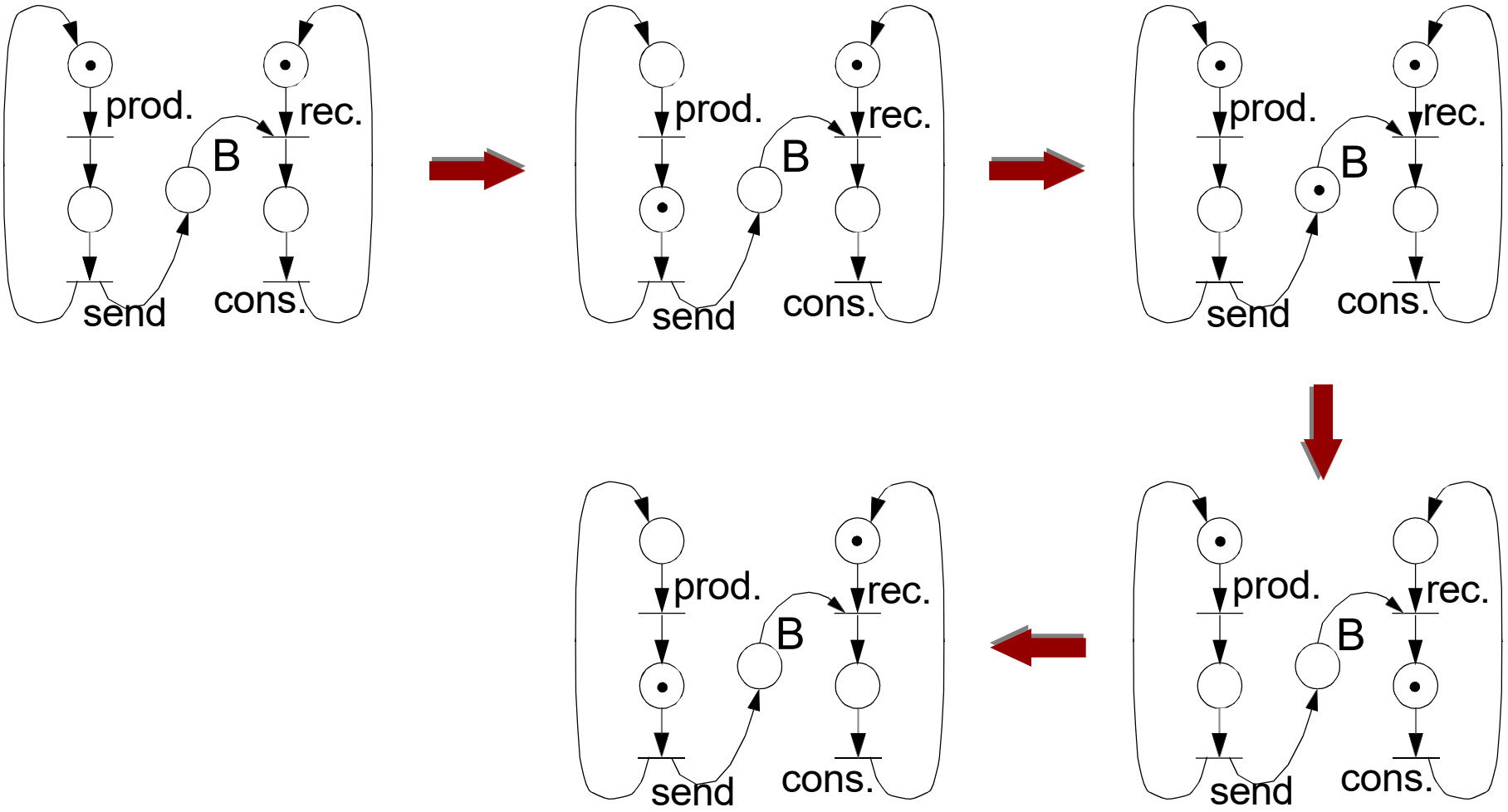

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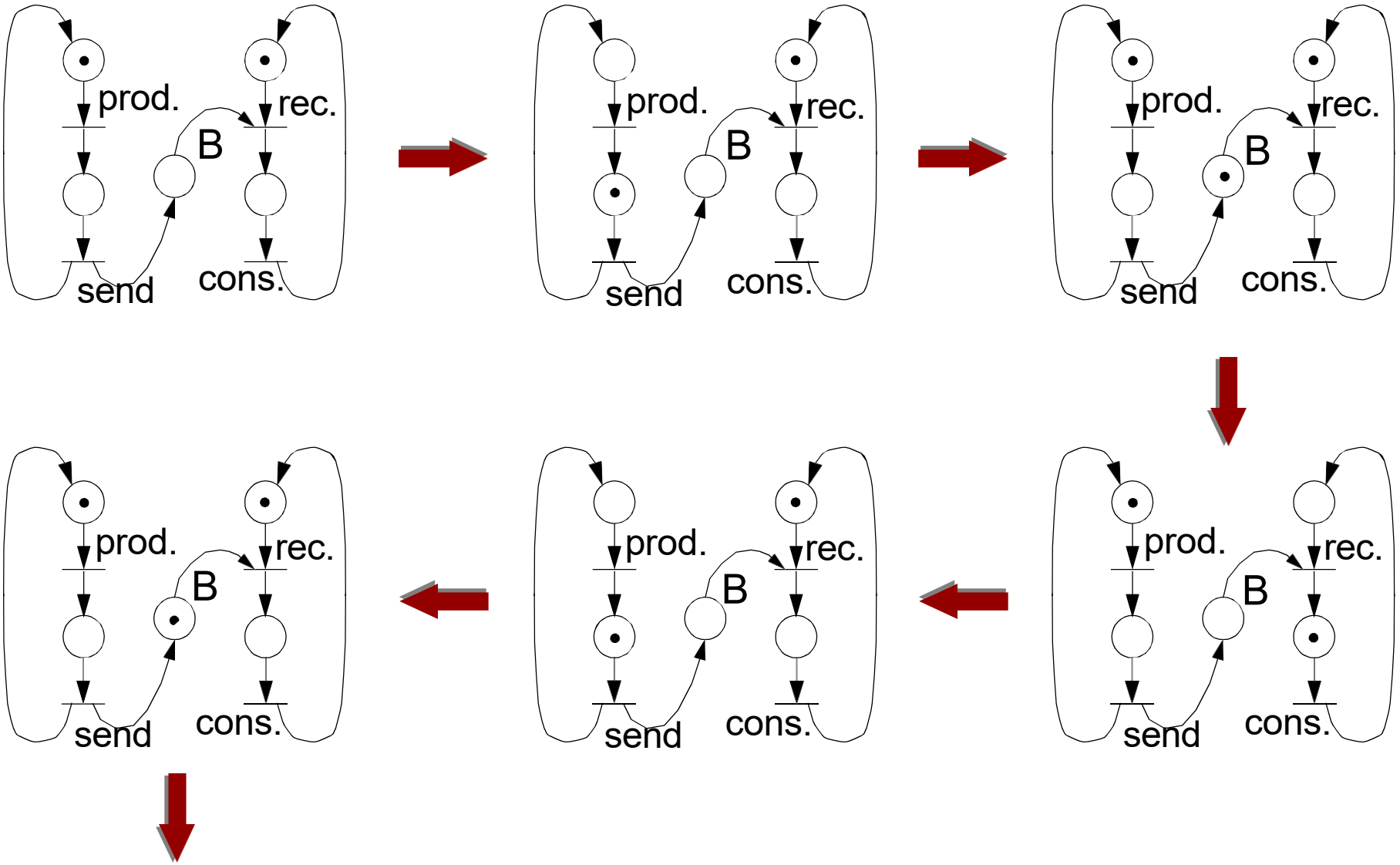

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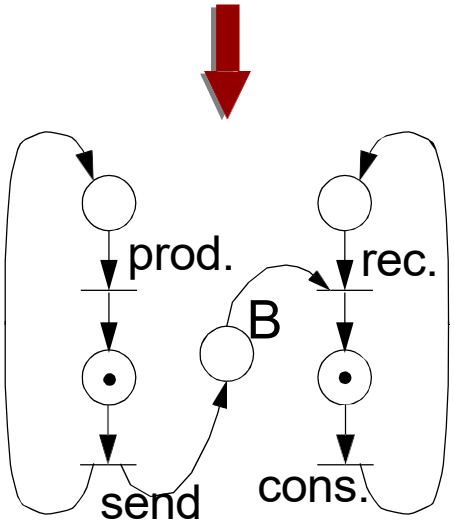

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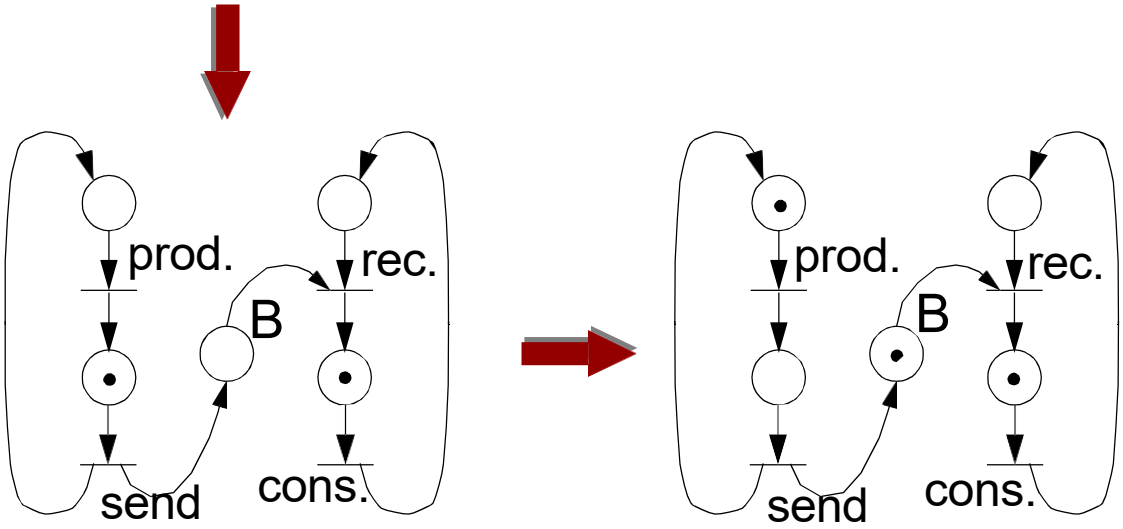

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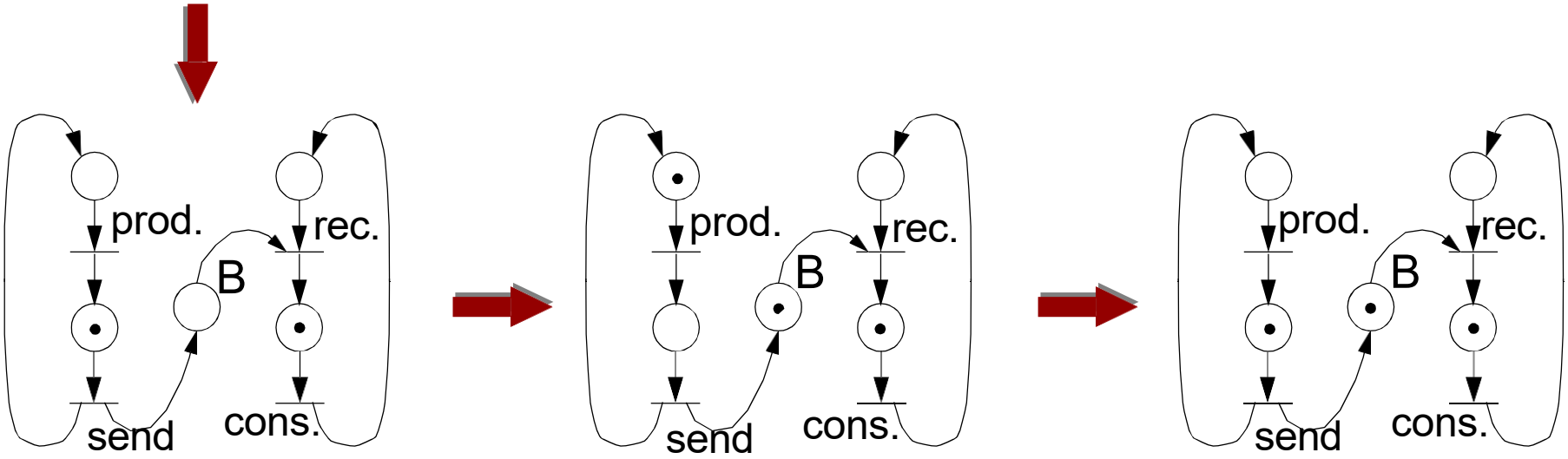

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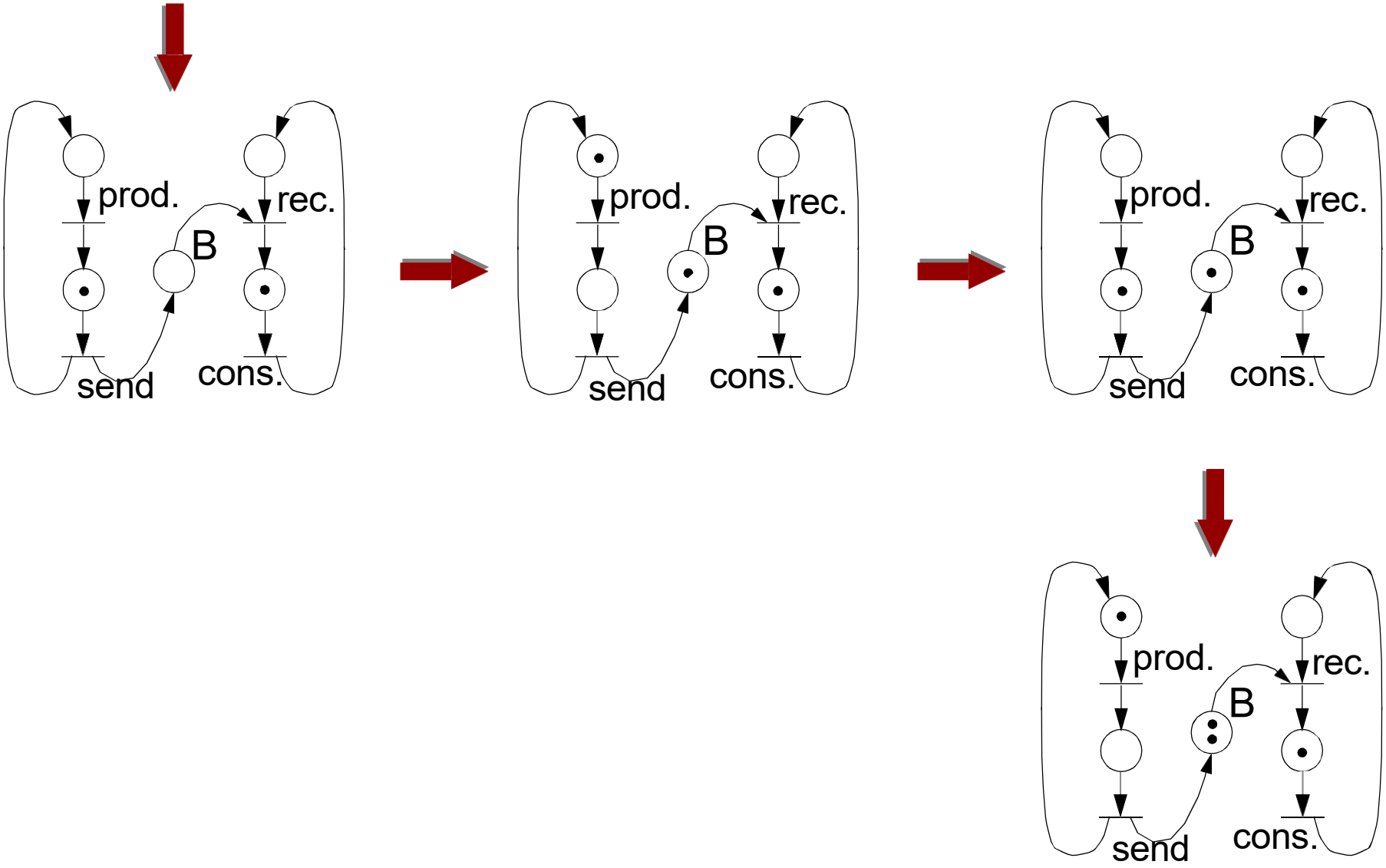

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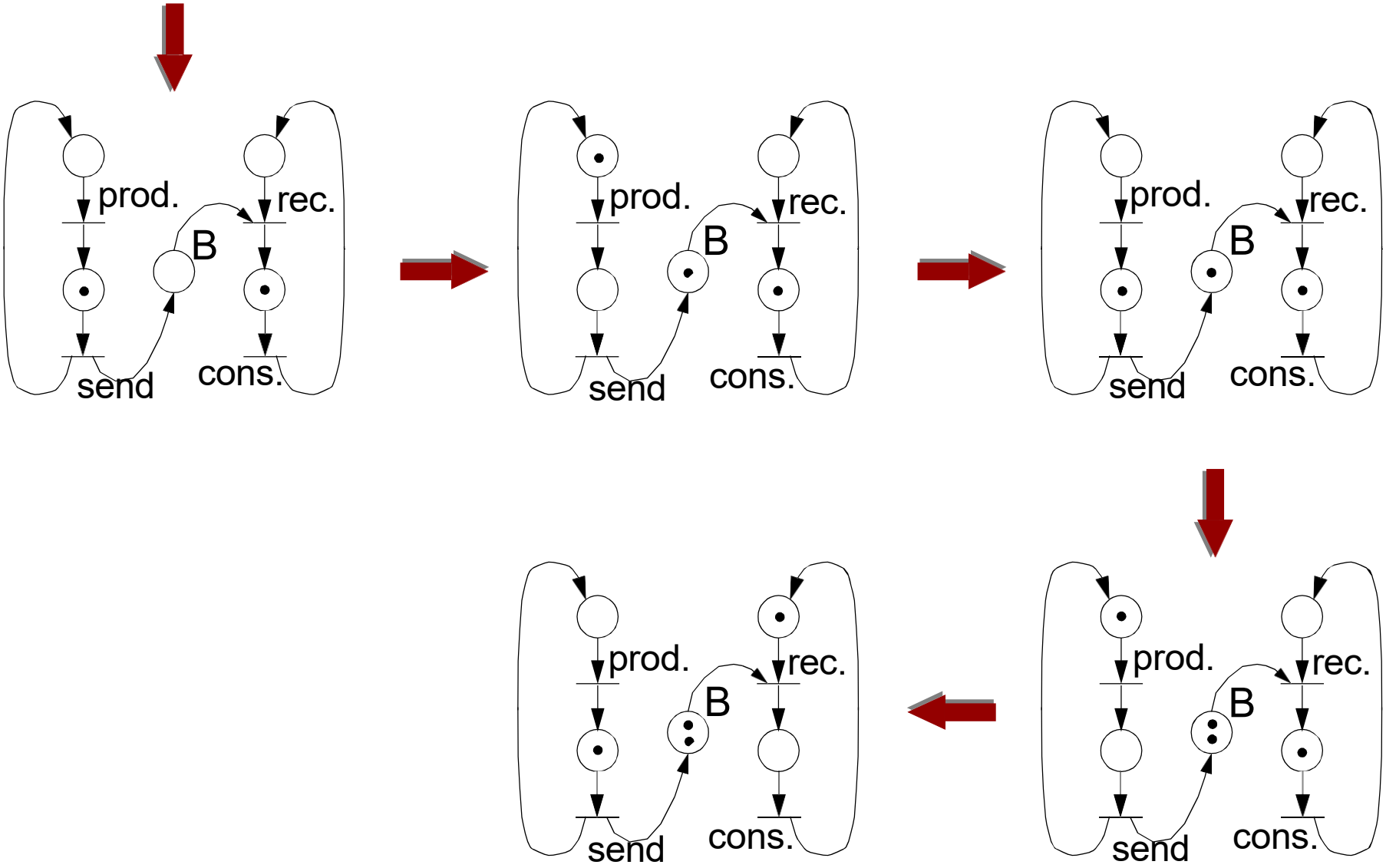

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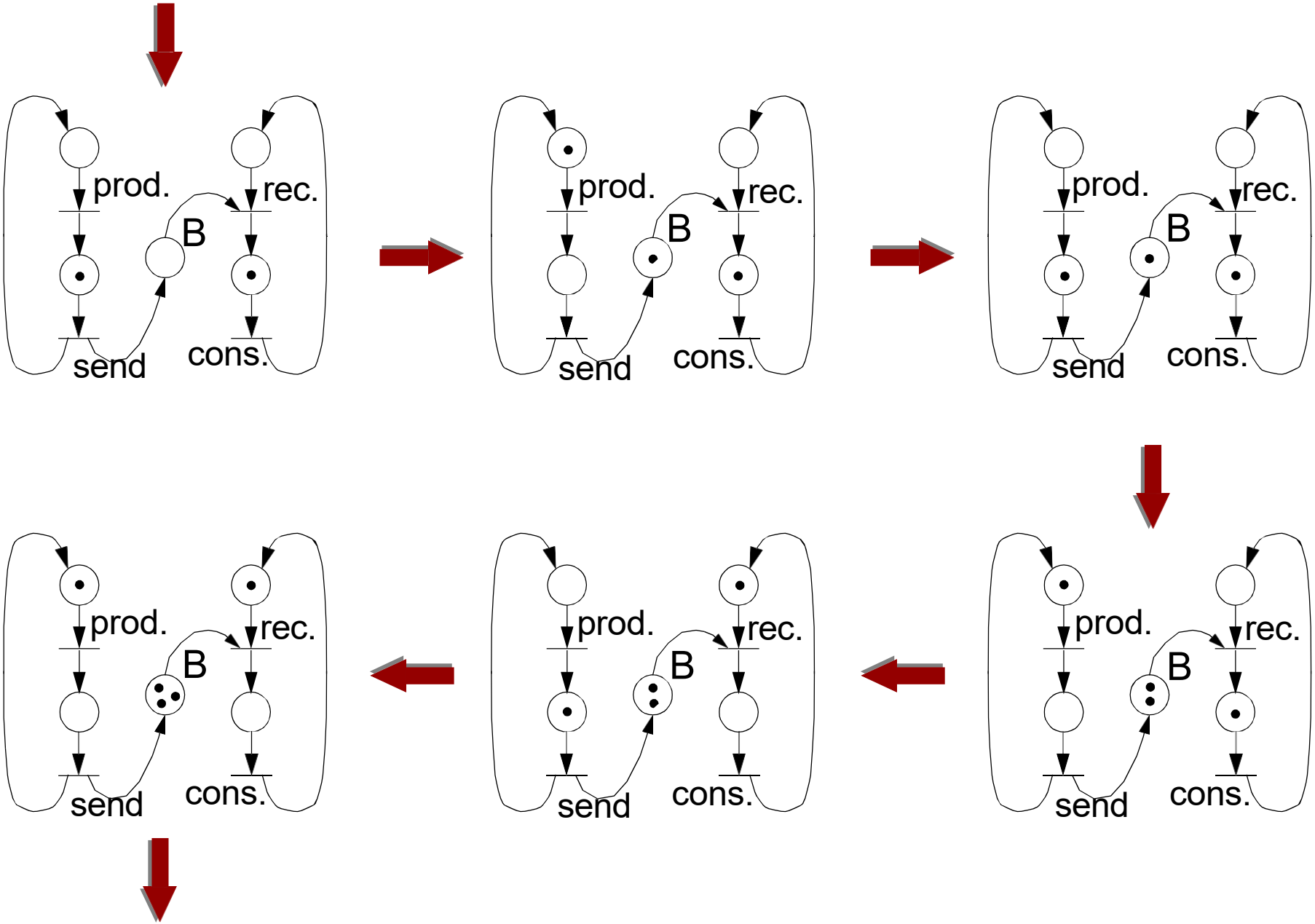

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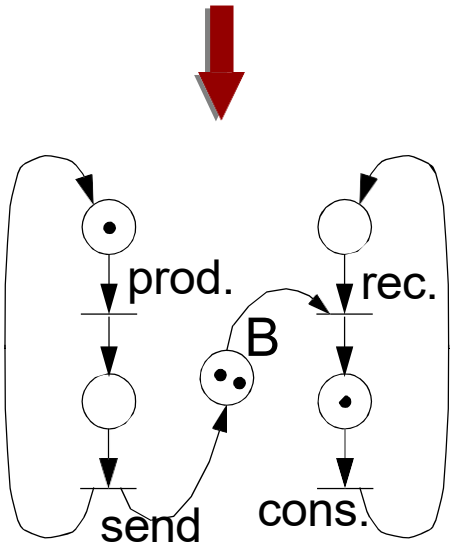

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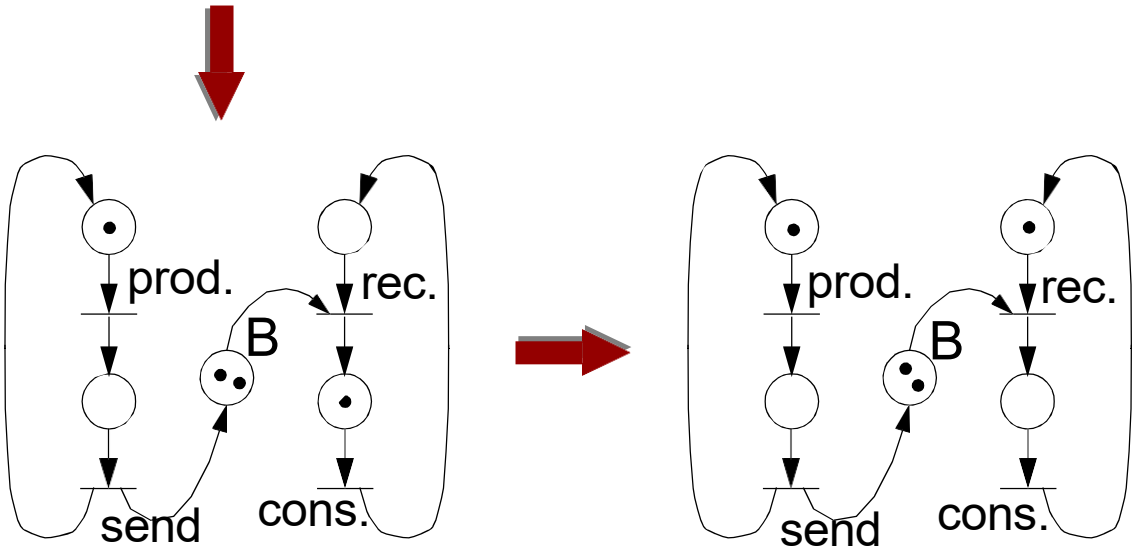

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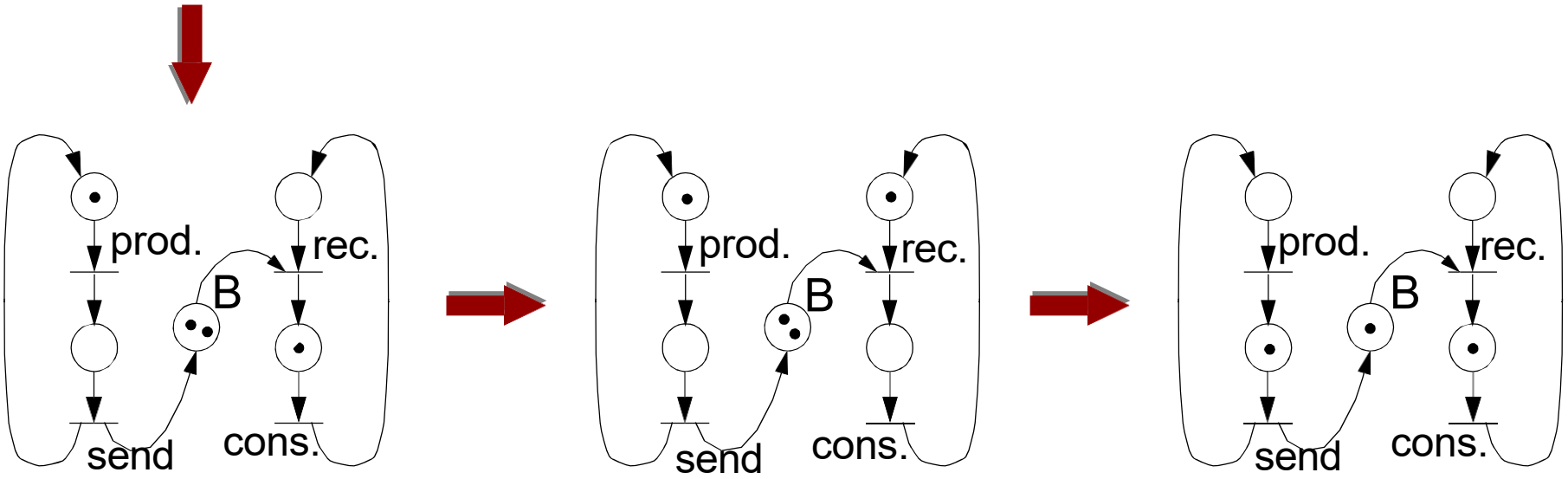

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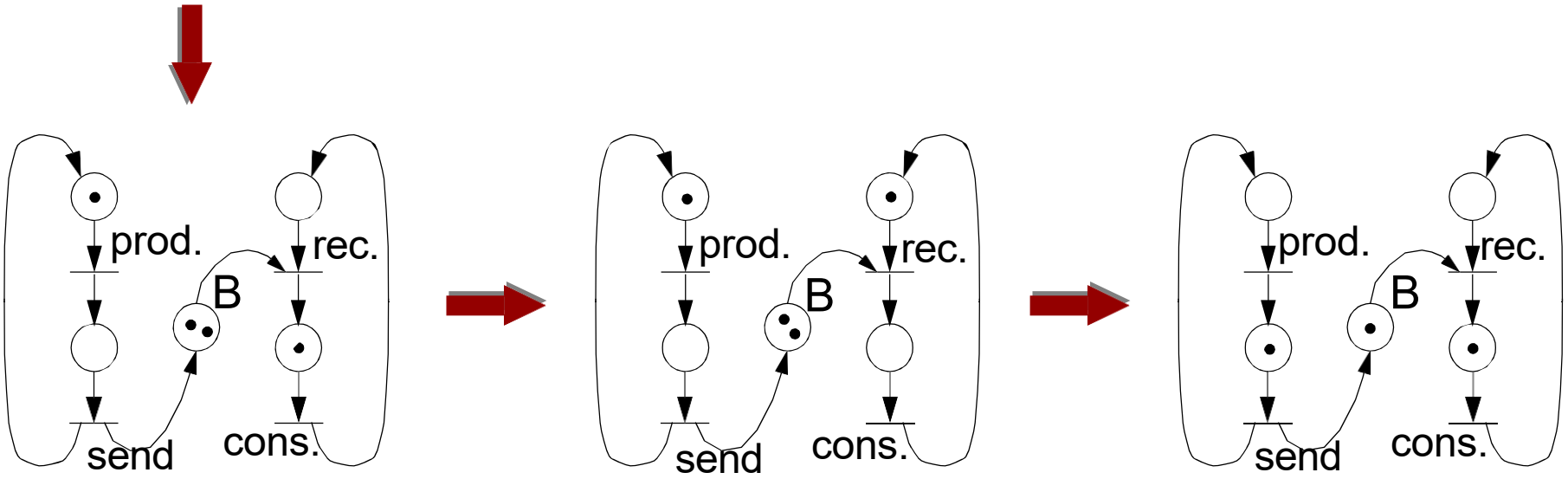
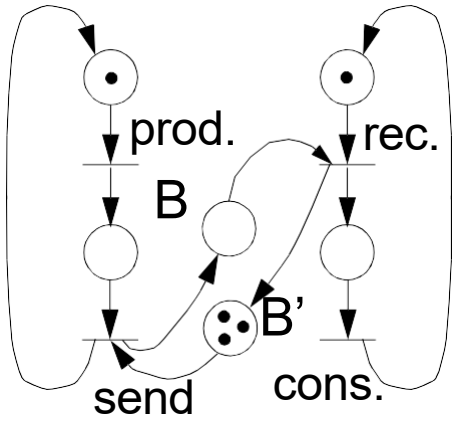


- Notice that the buffer is considered to be infinite (tokens accumulate in *B*).

Petri Net Example

Here we have the same model as on the previous slides, but with limited buffer. The buffer size is three (number of initial tokens in *B*')



- Nr. of tokens in B': how many free slots are available in the buffer;
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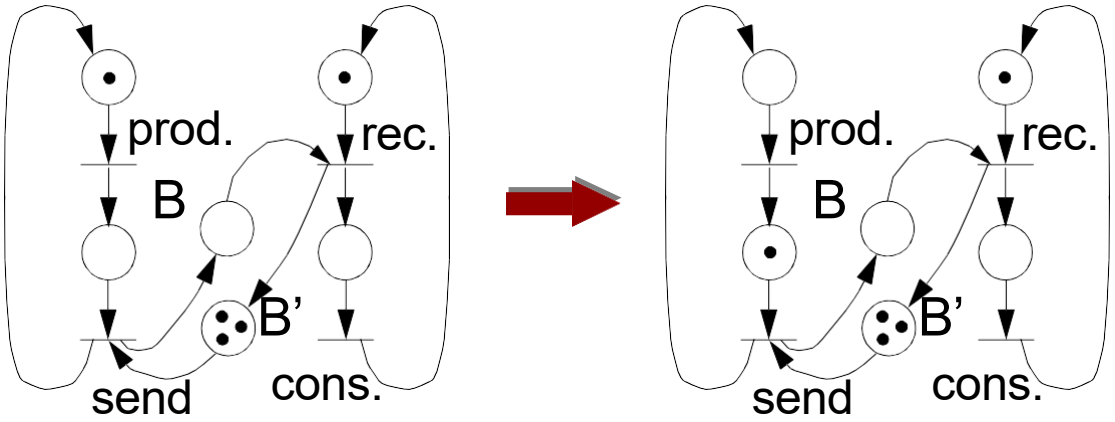

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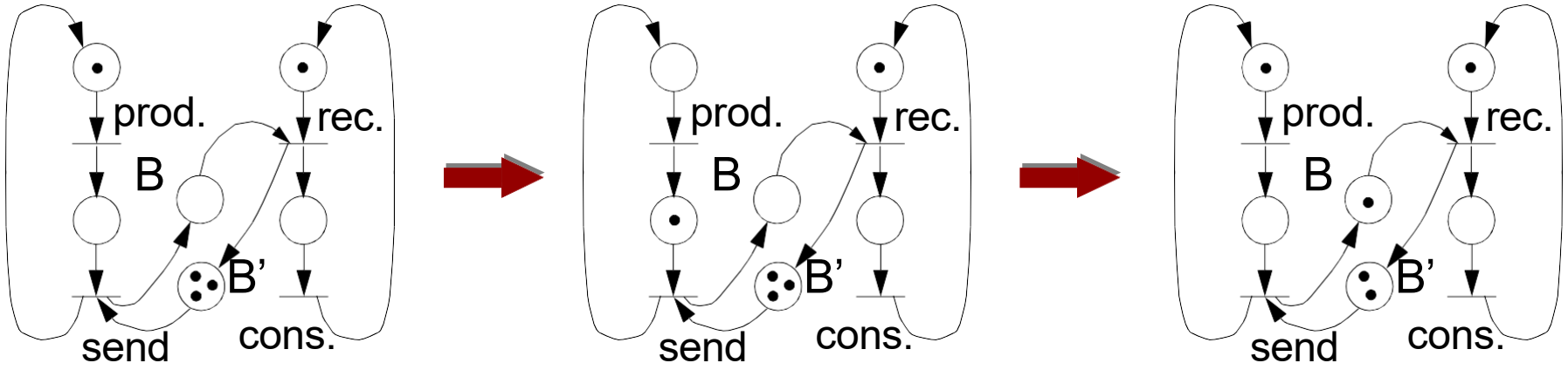

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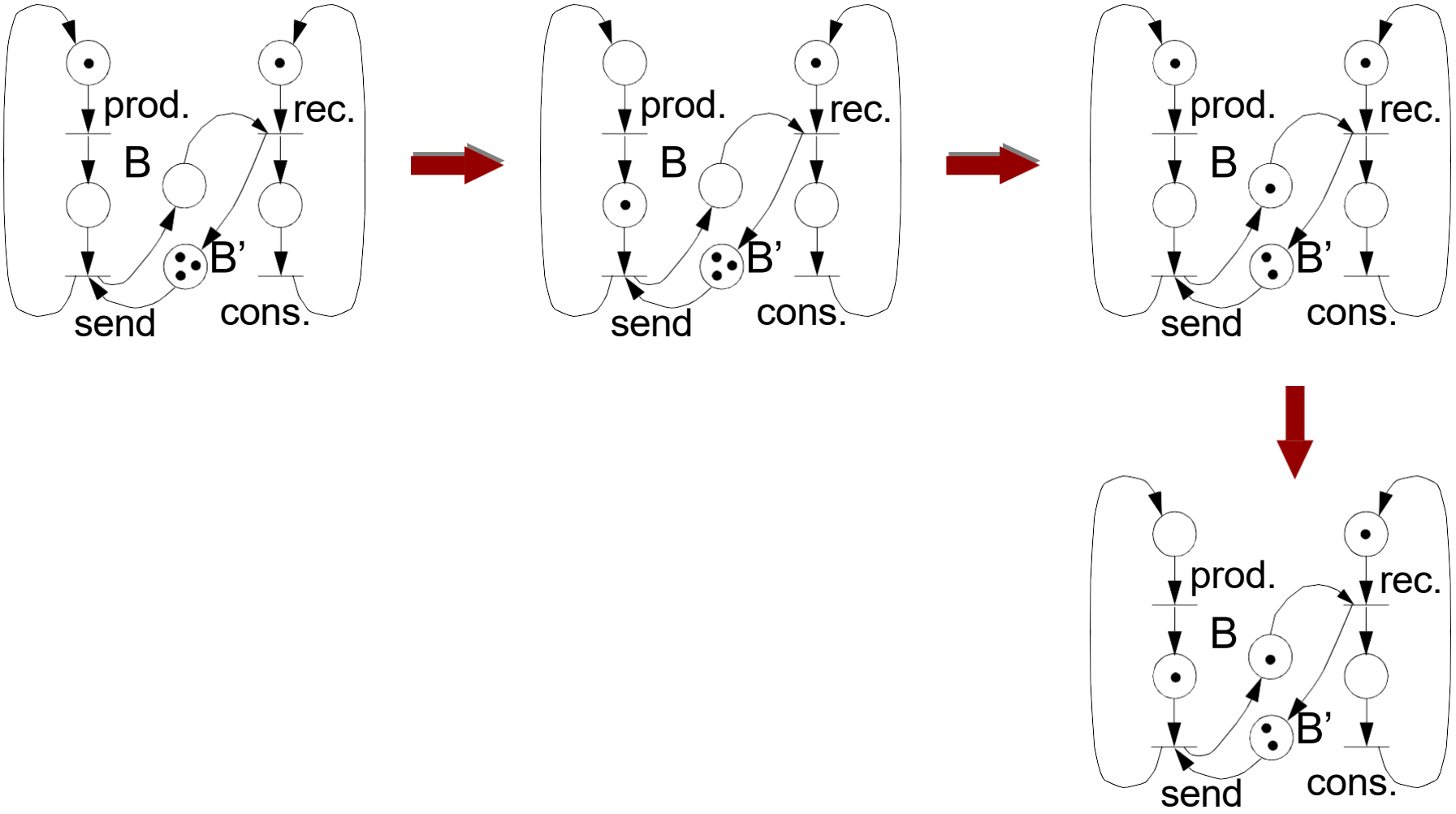

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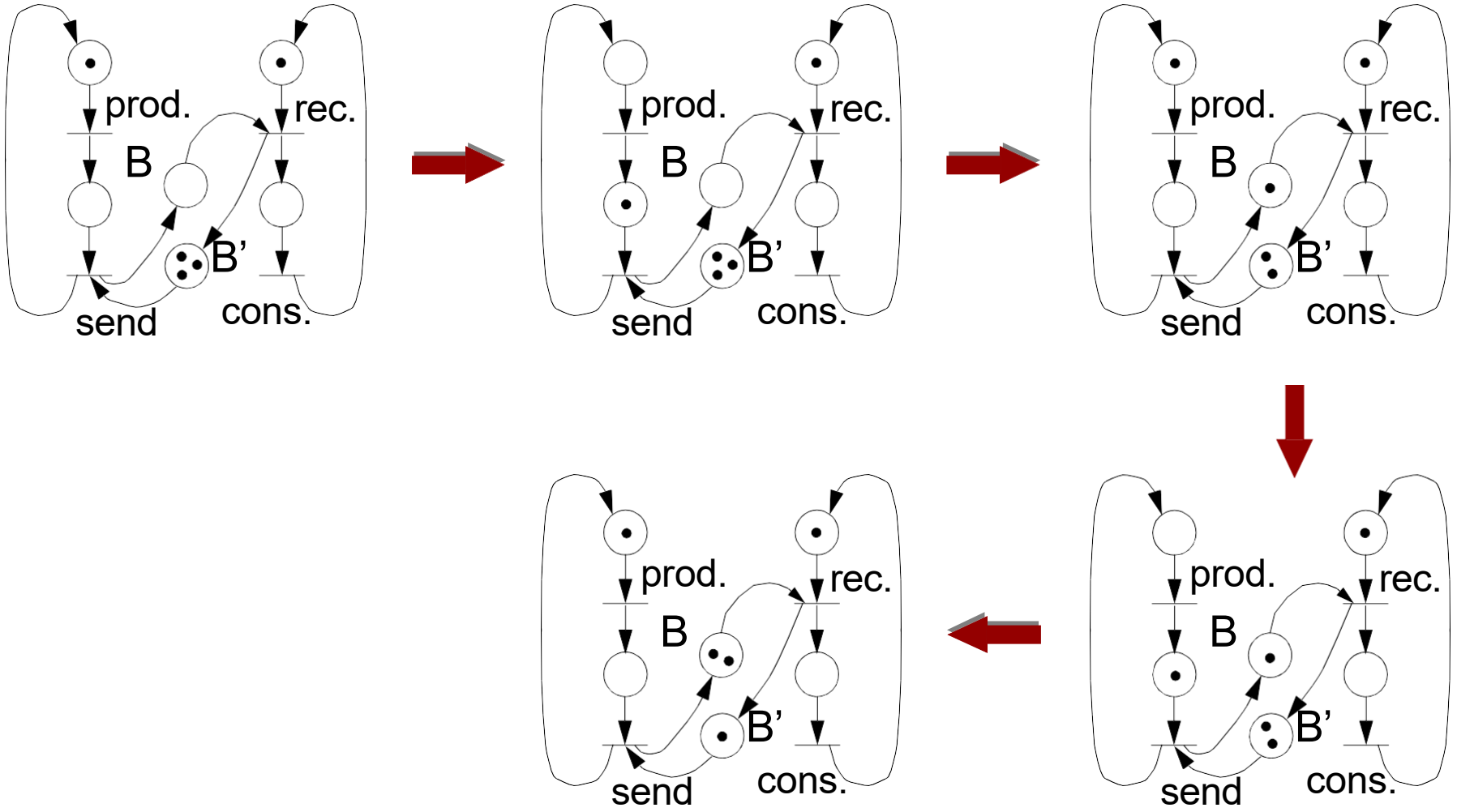

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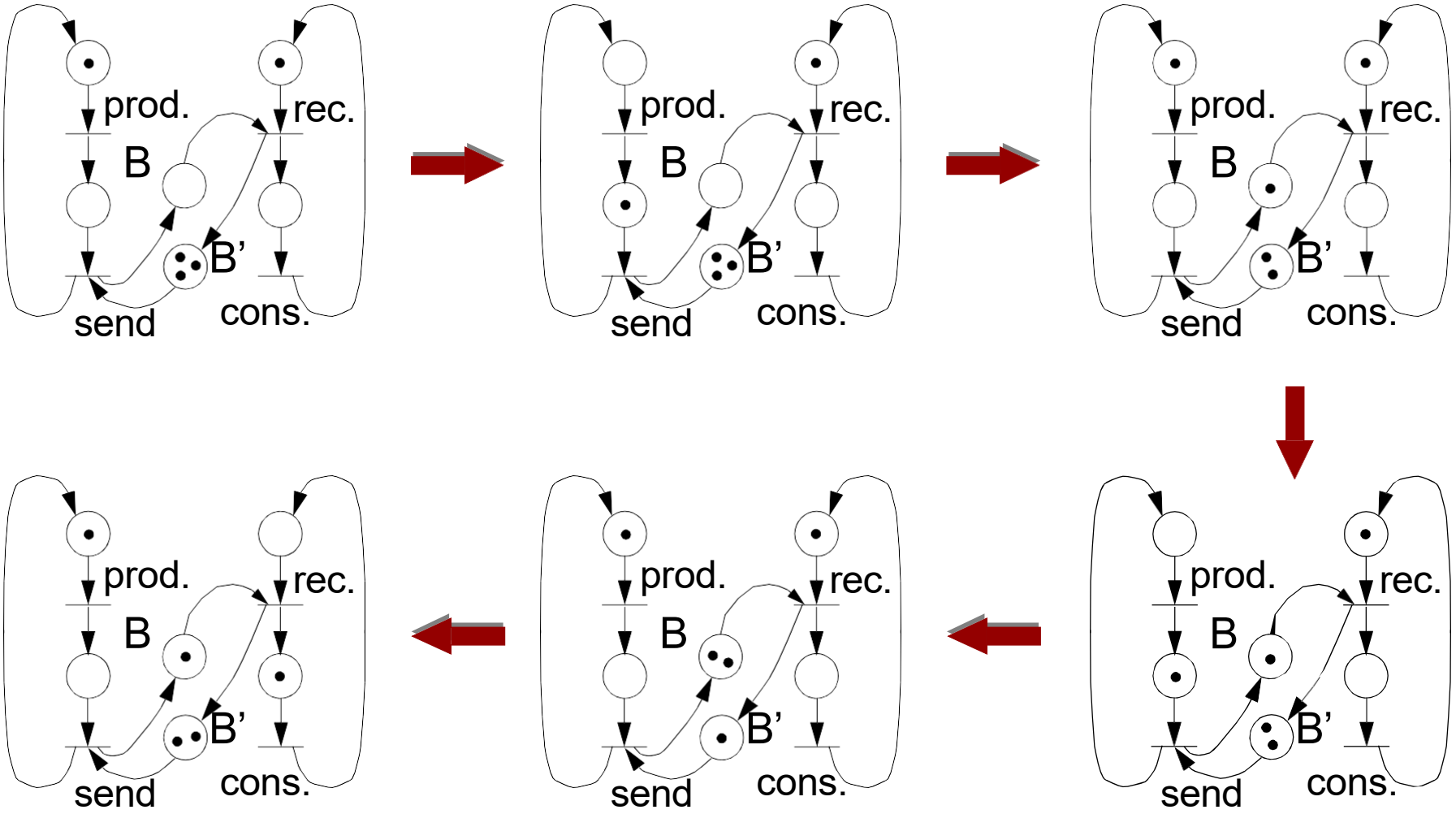

Some Features and Applications of Petri Nets

- Intuitive.

  Easy to express concurrency, synchronisation, nondeterminism.

  Nondeterminism is an important difference between Petri nets and dataflow!

- As an uninterpreted model, Petri Nets can be used for several, very different classes of problems.

  - *Uninterpreted model*: nothing has to be specified related to the particular activities associated to the transitions.

Some Features and Applications of Petri Nets

- Petri Nets have been intensively used for modeling and analysis of industrial production systems, information systems, but also
  - Computer architectures
  - Operating systems
  - Concurrent programs
  - Distributed systems
  - Hardware systems

Properties and Analysis of Petri Nets

- Several properties of the system can be analysed using Petri nets:
  - *Boundedness*: number of tokens in a place does not exceed a limit. If this limit is 1, the property is sometimes called *safeness*.
    - You can check that available resources are not exceeded.

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    The whole net is live, if all its transitions are live.
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    The whole net is live, if all its transitions are live.
    - Important in order to check that the system is not deadlocked.
  - *Reachability*: given a current marking M and another marking M', does there exist a sequence of transitions by which M' can be obtained?
    - You can check that a certain desired state (marking) is reached.
    - You can check that a certain undesired state is never reached.

Properties and Analysis of Petri Nets

Mathematical tools are available for analysis of Petri Nets.

The properties discussed above can be formally verified.

- Petri nets (like dataflow systems) are *asynchronous concurrent*.
  - Events can happen at any time.
  - There exists a partial order of events.

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  - Transitions have associated times (time intervals)
  - Tokens are carrying time stamps.

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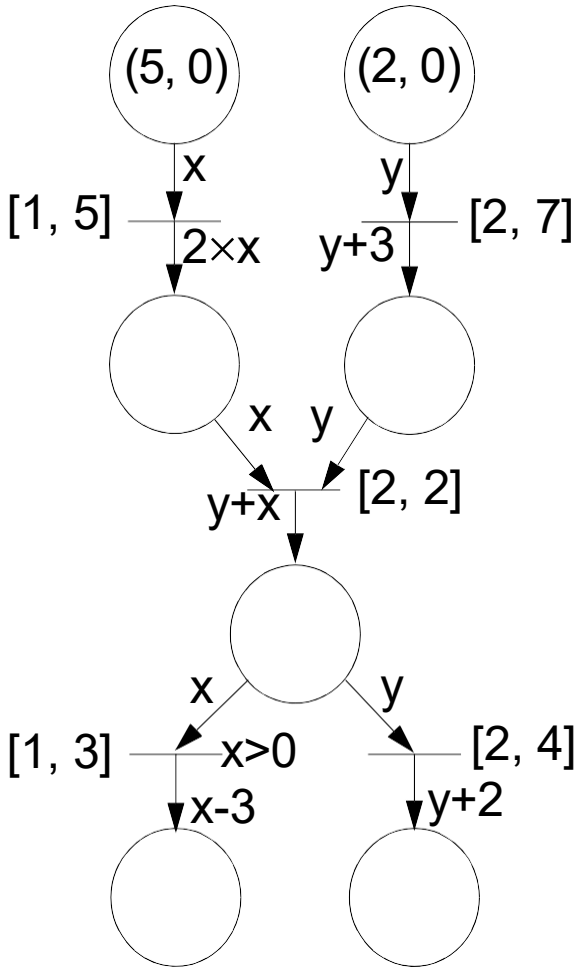
- Coloured Petri Nets
  - Tokens have associated values
  - Transitions have associated functions

  Coloured Petri Nets are similar to dataflow models (but also capture nondeterminism!).

Extended Petri Net Models

Coloured and Timed Petri net



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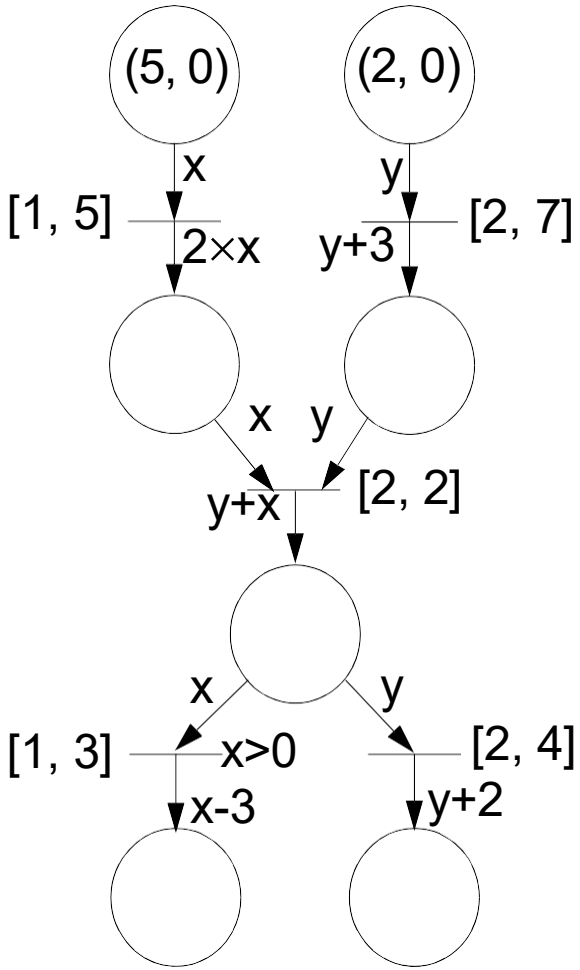

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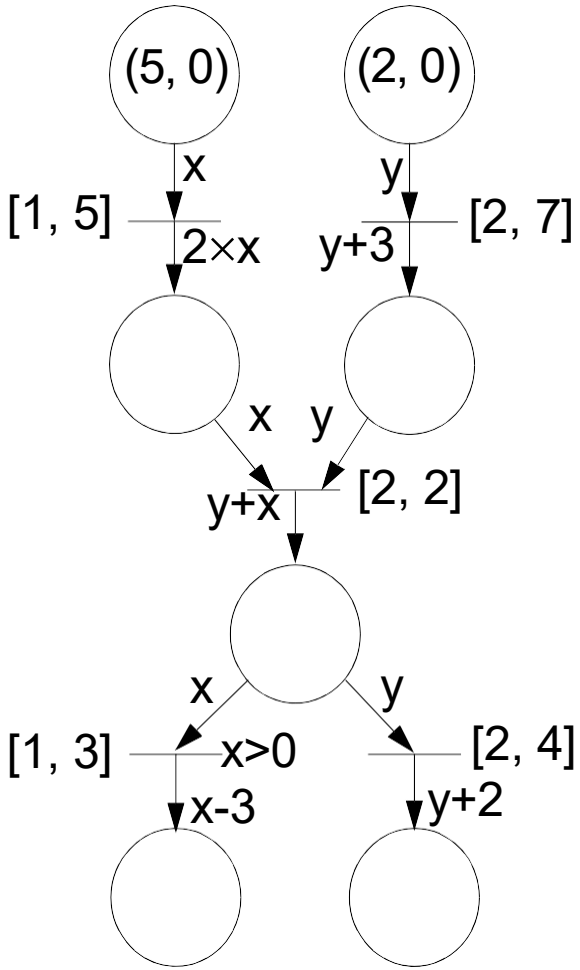
Extended Petri Net Models

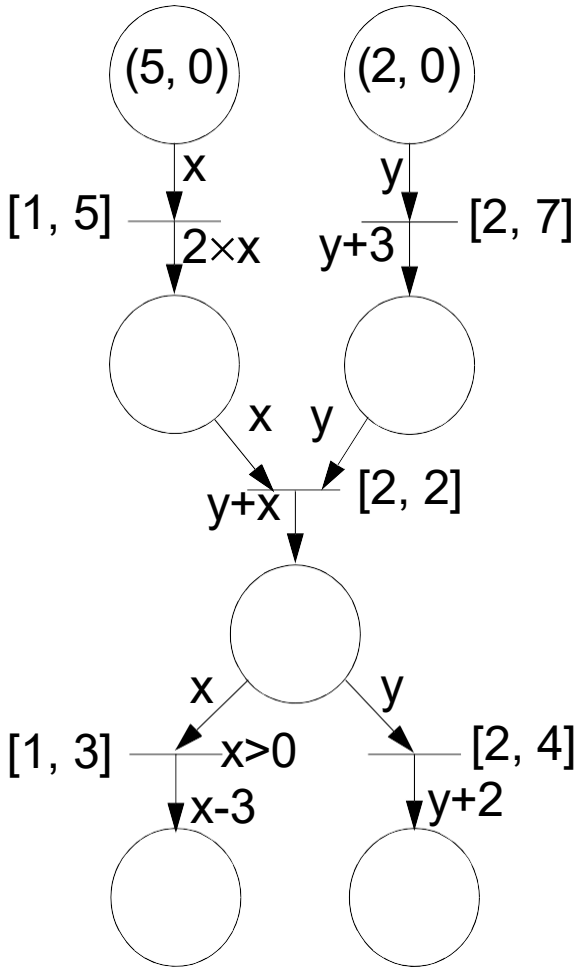
Coloured and Timed Petri net



- Tokens carry Time stamps
- Transitions have associated time (interval)
- Tokens have associated values

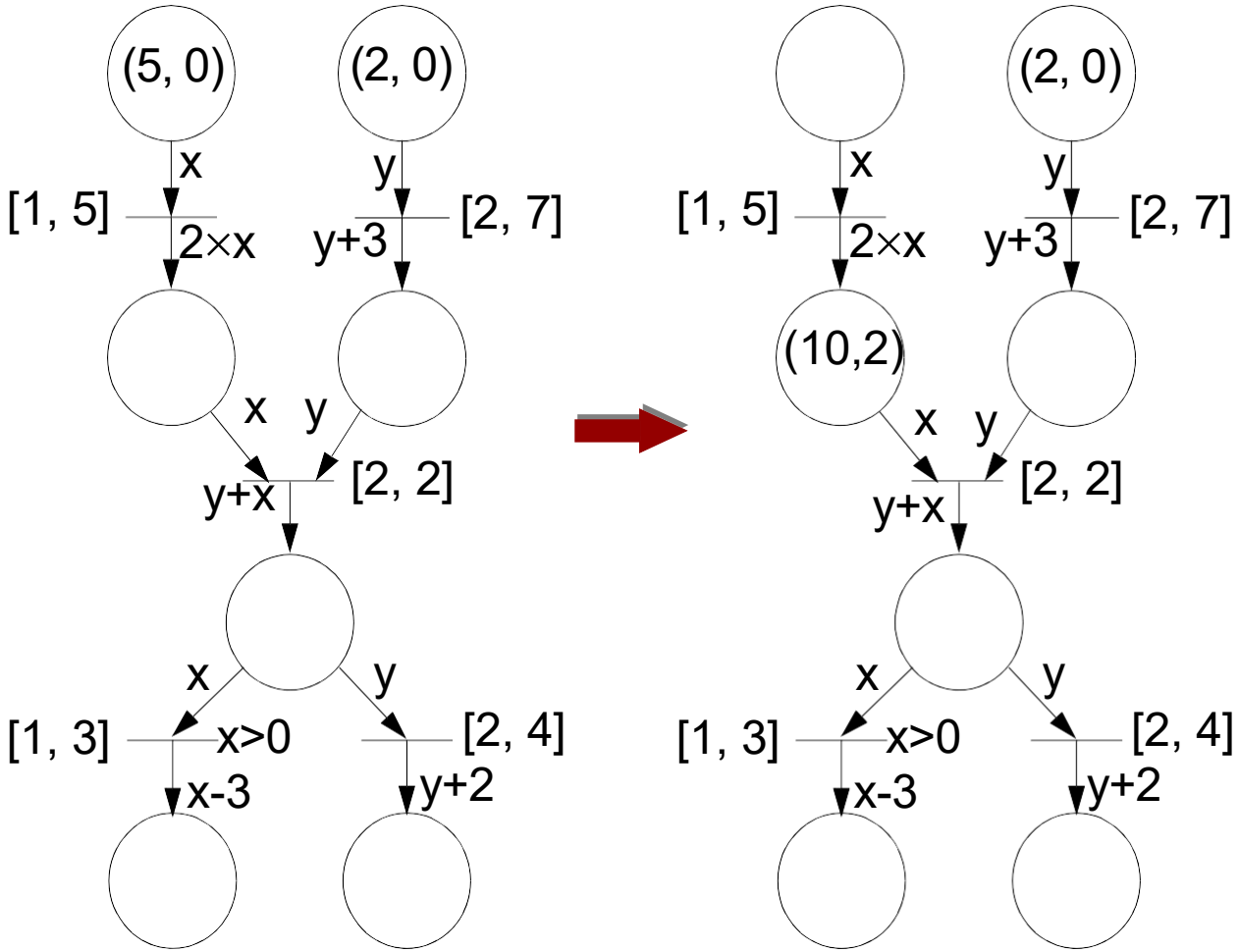
Extended Petri Net Models

Coloured and Timed Petri net

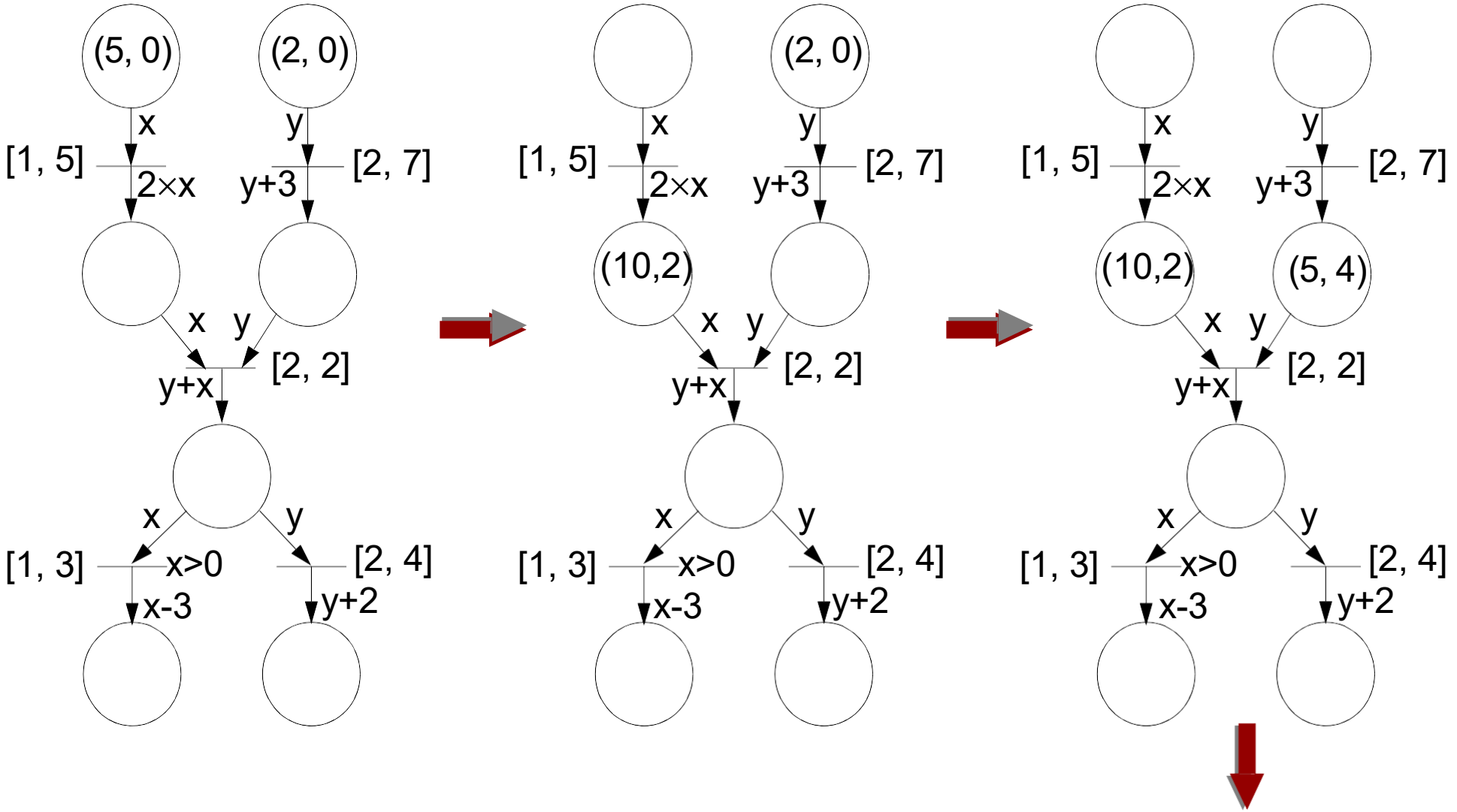


- Tokens carry Time stamps
- Transitions have associated time (interval)
- Tokens have associated values
- Transitions have associated functions and guards

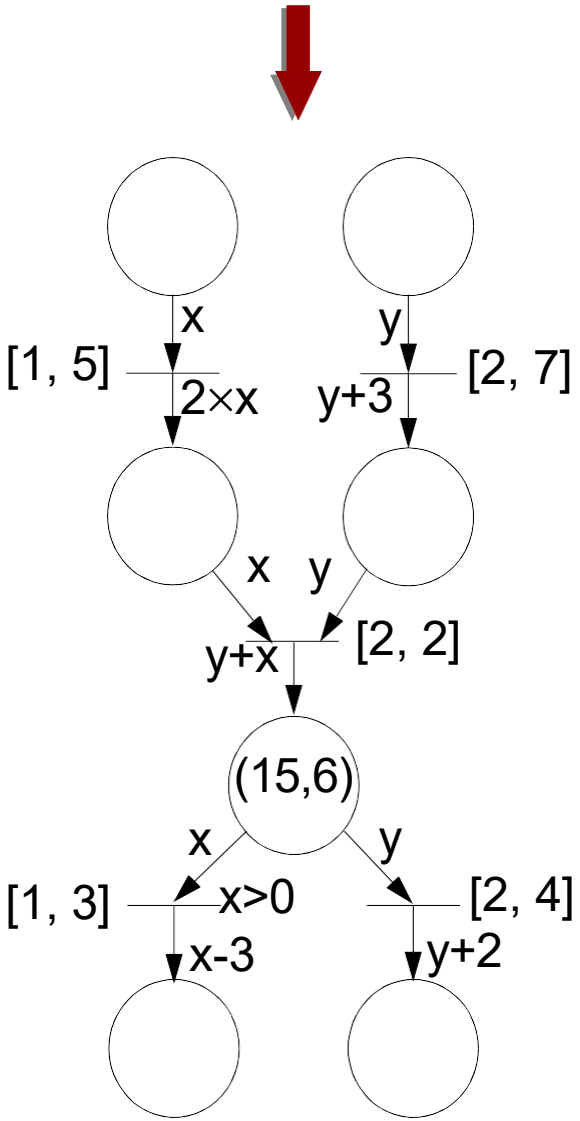
Extended Petri Net Models



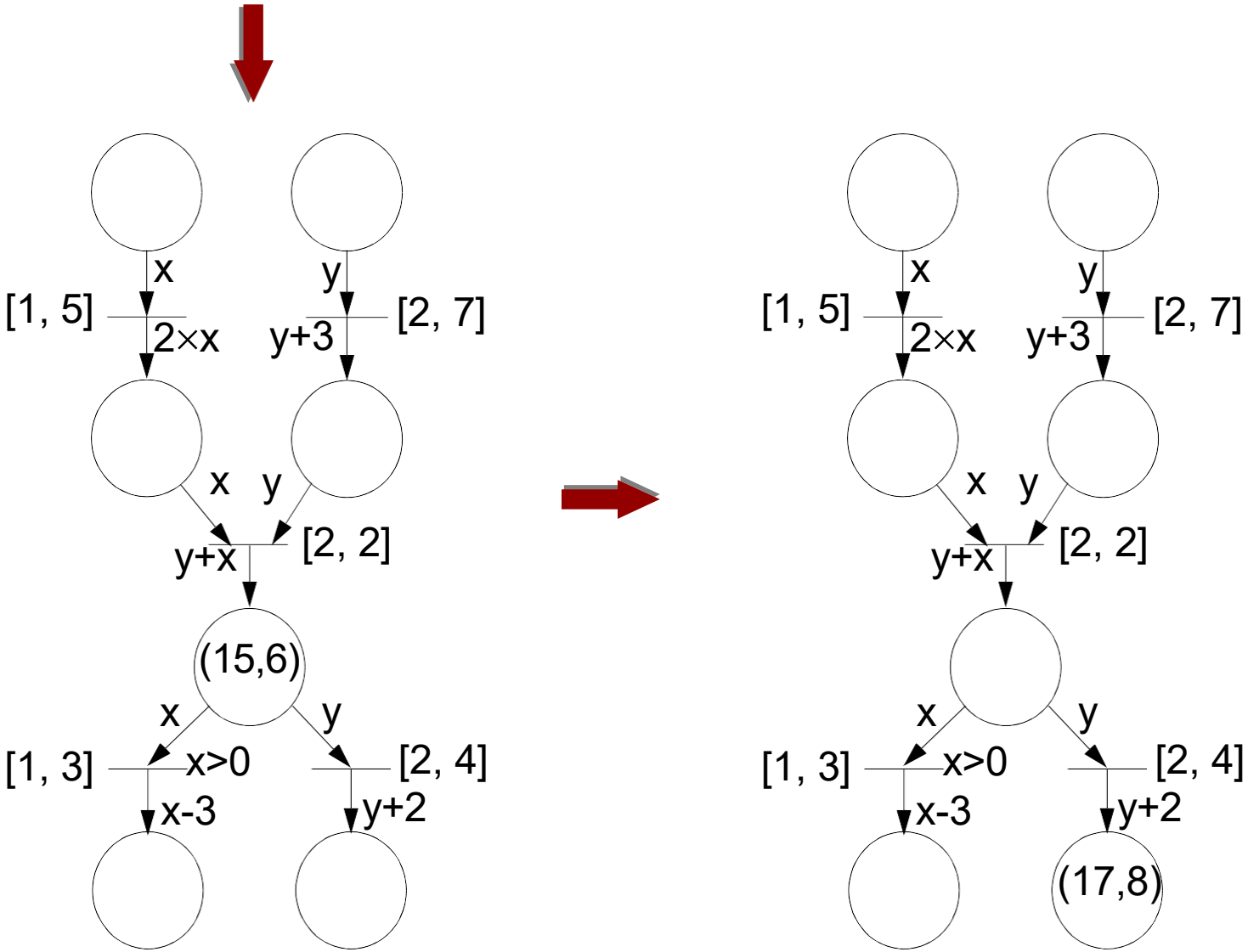
Extended Petri Net Models



Extended Petri Net Models



Extended Petri Net Models



Extended Petri Net Models

- Extended Petri Nets have a larger expressive power then classical Petri Nets.

Analysis is more complex; the formal analysis of properties can take very large amounts of time (memory).

- Simulation of the Petri Net is very often used in order to verify the system and to estimate performance